扩展卡尔曼滤波（EKF）

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卡尔曼滤波与组合导航原理(4-6讲)

卡尔曼滤波与组合导航原理

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$$f(x)=f(x_0)+\frac{1}{1!}f'(x_0)(x-x_0)+\frac{1}{2!}f''(x_0)(x-x_0)^2+\cdots$$

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13.1 多元向量函数的泰勒级数展开

$$\boldsymbol{Y}=\boldsymbol{f}(\boldsymbol{X})$$

$$\begin{cases} y_1=f_1(\boldsymbol{X})=f_1(x_1,x_2,\cdots x_n) \\ y_2=f_2(\boldsymbol{X})=f_2(x_1,x_2,\cdots x_n) \\ \vdots \\ y_m=f_m(\boldsymbol{X})=f_m(x_1,x_2,\cdots x_n) \end{cases}$$

泰勒级数展开式

$$\boldsymbol{Y}=\boldsymbol{f}(\boldsymbol{X}^{(0)})+\sum_{i=1}^{\infty}\frac{1}{i!}(\nabla^{\mathrm{T}}\cdot\delta\boldsymbol{X})^i\boldsymbol{f}(\boldsymbol{X}^{(0)})$$

其中 $\delta\boldsymbol{X}=\boldsymbol{X}-\boldsymbol{X}^{(0)}$

$$\nabla=\begin{bmatrix}\frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \cdots & \frac{\partial}{\partial x_n}\end{bmatrix}^{\mathrm{T}}$$

一阶展开项的详细表达式

$$\frac{1}{1}(\nabla^{\mathrm{T}}\cdot\delta\boldsymbol{X})^1\boldsymbol{f}(\boldsymbol{X}^{(0)})=\left(\frac{\partial}{\partial x_1}\delta x_1+\frac{\partial}{\partial x_2}\delta x_2+\cdots+\frac{\partial}{\partial x_n}\delta x_n\right)\boldsymbol{f}(\boldsymbol{X})\bigg|_{X=X^{(0)}}$$

$$=\begin{bmatrix}\frac{\partial f_1(\boldsymbol{X})}{\partial x_1}\delta x_1+\frac{\partial f_1(\boldsymbol{X})}{\partial x_2}\delta x_2+\cdots+\frac{\partial f_1(\boldsymbol{X})}{\partial x_n}\delta x_n \\ \frac{\partial f_2(\boldsymbol{X})}{\partial x_1}\delta x_1+\frac{\partial f_2(\boldsymbol{X})}{\partial x_2}\delta x_2+\cdots+\frac{\partial f_2(\boldsymbol{X})}{\partial x_n}\delta x_n \\ \vdots \\ \frac{\partial f_m(\boldsymbol{X})}{\partial x_1}\delta x_1+\frac{\partial f_m(\boldsymbol{X})}{\partial x_2}\delta x_2+\cdots+\frac{\partial f_m(\boldsymbol{X})}{\partial x_n}\delta x_n\end{bmatrix}_{X=X^{(0)}}=\begin{bmatrix}\frac{\partial f_1(\boldsymbol{X})}{\partial x_1} & \frac{\partial f_1(\boldsymbol{X})}{\partial x_2} & \cdots & \frac{\partial f_1(\boldsymbol{X})}{\partial x_n} \\ \frac{\partial f_2(\boldsymbol{X})}{\partial x_1} & \frac{\partial f_2(\boldsymbol{X})}{\partial x_2} & \cdots & \frac{\partial f_2(\boldsymbol{X})}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m(\boldsymbol{X})}{\partial x_1} & \frac{\partial f_m(\boldsymbol{X})}{\partial x_2} & \cdots & \frac{\partial f_m(\boldsymbol{X})}{\partial x_n}\end{bmatrix}\begin{bmatrix}\delta x_1 \\ \delta x_2 \\ \vdots \\ \delta x_n\end{bmatrix}_{X=X^{(0)}}$$

$$=J(\boldsymbol{f}(\boldsymbol{X}))\delta\boldsymbol{X}\big|_{X=X^{(0)}}$$

↳偏差向量

$$J(\boldsymbol{f}(\boldsymbol{X}))=\frac{\partial\boldsymbol{f}(\boldsymbol{X})}{\partial\boldsymbol{X}^{\mathrm{T}}}$$ Jacobian矩阵

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二阶展开项的详细表达式

$$Y = f(X^{(0)}) + \sum_{i=1}^{\infty} \frac{1}{i!}(\nabla^{\mathrm{T}} \cdot \delta X)^i f(X^{(0)})$$

$$\frac{1}{2}(\nabla^{\mathrm{T}} \cdot \delta X)^2 f(X^{(0)}) = \frac{1}{2}\left(\frac{\partial}{\partial x_1}\delta x_1 + \frac{\partial}{\partial x_2}\delta x_2 + \cdots + \frac{\partial}{\partial x_n}\delta x_n\right)^2 f(X)\bigg|_{X=X^{(0)}}$$

$$= \frac{1}{2}\mathrm{tr}\left(\begin{bmatrix} \frac{\partial^2}{\partial x_1^2} & \frac{\partial^2}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2}{\partial x_1 \partial x_n} \\ \frac{\partial^2}{\partial x_2 \partial x_1} & \frac{\partial^2}{\partial x_2^2} & \cdots & \frac{\partial^2}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2}{\partial x_n \partial x_1} & \frac{\partial^2}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2}{\partial x_n^2} \end{bmatrix} \begin{bmatrix} \delta x_1^2 & \delta x_1 \delta x_2 & \cdots & \delta x_1 \delta x_n \\ \delta x_2 \delta x_1 & \delta x_2^2 & \cdots & \delta x_2 \delta x_n \\ \vdots & \vdots & \ddots & \vdots \\ \delta x_n \delta x_1 & \delta x_n \delta x_2 & \cdots & \delta x_n^2 \end{bmatrix}\right) f(X)\Bigg|_{X=X^{(0)}}$$

推导过程自己慢慢看吧。

$$= \frac{1}{2}\mathrm{tr}(\nabla\nabla^{\mathrm{T}} \cdot \delta X \delta X^{\mathrm{T}}) f(X)\Big|_{X=X^{(0)}} = \frac{1}{2}\mathrm{tr}(\nabla\nabla^{\mathrm{T}} \cdot \delta X \delta X^{\mathrm{T}}) \sum_{j=1}^{m} e_j f_j(X)\Big|_{X=X^{(0)}}$$

第j元素为1的单位列向量

$$= \frac{1}{2}\sum_{j=1}^{m} e_j \mathrm{tr}(\nabla\nabla^{\mathrm{T}} \cdot \delta X \delta X^{\mathrm{T}}) f_j(X)\Big|_{X=X^{(0)}} = \frac{1}{2}\sum_{j=1}^{m} e_j \mathrm{tr}\left(\nabla\nabla^{\mathrm{T}} f_j(X)\big|_{X=X^{(0)}} \cdot \delta X \delta X^{\mathrm{T}}\right)$$

$$= \frac{1}{2}\sum_{j=1}^{m} e_j \mathrm{tr}\left(\mathscr{H}\left(f_j(X)\right)\big|_{X=X^{(0)}} \cdot \delta X \delta X^{\mathrm{T}}\right)$$

Hessian矩阵

得泰勒展开表示

$$Y = f(X^{(0)}) + J(f(X^{(0)}))\delta X + \frac{1}{2}\sum_{i=1}^{m} e_i \mathrm{tr}(\mathscr{H}(f_i(X^{(0)})) \cdot \delta X \delta X^{\mathrm{T}}) + O^3$$

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针对非线性 → 线性化：一阶展开近似忽略高阶。

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13.2 EKF滤波

一般形式 $X_k = f(X_{k-1}, W_{k-1}, k-1)$

状态空间模型(加性噪声)
$$\begin{cases} X_k = f(X_{k-1}) + \Gamma_{k-1} W_{k-1} \\ Z_k = h(X_k) + V_k \end{cases}$$

?没理解

怎么取? ←

k-1时刻参考值 X_{k-1}^n 偏差 $\delta X_{k-1} = X_{k-1} - X_{k-1}^n$

状态一步预测 $X_{k/k-1}^n = f(X_{k-1}^n)$ 偏差 $\delta X_k = X_k - X_{k/k-1}^n$

量测一步预测 $Z_{k/k-1}^n = h(X_{k/k-1}^n)$ 偏差 $\delta Z_k = Z_k - Z_{k/k-1}^n$

状态偏差方程

$$X_k \approx f(X_{k-1}^n) + J(f(X_{k-1}^n))(X_{k-1} - X_{k-1}^n) + \Gamma_{k-1} W_{k-1}$$ → 在参考点展开

$$X_k - f(X_{k-1}^n) \approx \Phi_{k/k-1}^n (X_{k-1} - X_{k-1}^n) + \Gamma_{k-1} W_{k-1}$$

$$\delta X_k = \Phi_{k/k-1}^n \delta X_{k-1} + \Gamma_{k-1} W_{k-1}$$

量测偏差方程

$$Z_k \approx h(X_{k/k-1}^n) + J(h(X_{k/k-1}^n))(X_k - X_{k/k-1}^n) + V_k$$

同理

$$\delta Z_k = H_k^n \delta X_k + V_k$$

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偏差状态空间模型
$$\begin{cases}\delta X_k = \Phi^n_{k/k-1}\delta X_{k-1} + \Gamma_{k-1}W_{k-1} \\ \delta Z_k = H^n_k \delta X_k + V_k\end{cases}$$

滤波更新方程 $\delta\hat{X}_k = \Phi^n_{k/k-1}\delta\hat{X}_{k-1} + K^n_k(\delta Z_k - H^n_k\Phi^n_{k/k-1}\delta\hat{X}_{k-1})$

$$\xrightarrow[\delta\hat{X}_k = \hat{X}_k - X^n_{k/k-1}]{\delta\hat{X}_{k-1} = \hat{X}_{k-1} - X^n_{k-1}} \hat{X}_k = X^n_{k/k-1} + K^n_k(Z_k - Z^n_{k/k-1}) + (I - K^n_k H^n_k)\Phi^n_{k/k-1}(\hat{X}_{k-1} - X^n_{k-1})$$

→随便取 但选为上一时刻的估计值 ⇓ =0

$$\xrightarrow{X^n_{k-1} = \hat{X}_{k-1}} \quad = X^n_{k/k-1} + K^n_k(Z_k - Z^n_{k/k-1})$$

最后，得EKF滤波公式

$$\begin{cases}\hat{X}_{k/k-1} = f(\hat{X}_{k-1}) \\ P_{k/k-1} = \Phi_{k/k-1}P_{k-1}\Phi^{\mathrm{T}}_{k/k-1} + \Gamma_{k-1}Q_{k-1}\Gamma^{\mathrm{T}}_{k-1} \\ K_k = P_{k/k-1}H^{\mathrm{T}}_k(H_kP_{k/k-1}H^{\mathrm{T}}_k + R_k)^{-1} \\ \hat{X}_k = \hat{X}_{k/k-1} + K_k\left[Z_k - h(\hat{X}_{k/k-1})\right] \\ P_k = (I - K_kH_k)P_{k/k-1}\end{cases} \qquad \begin{aligned}\Phi_{k/k-1} &= J(f(\hat{X}_{k-1})) \\ H_k &= J(h(\hat{X}_{k/k-1}))\end{aligned}$$

⇒ 状态方程 ⇒ 偏差方程 ⇓ EKF ⇐ 代回偏差

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13.3 二阶滤波 →思路和EKF一样

状态方程的二阶泰勒级数展开 (Hessian矩阵)

$$X_k = f(X^n_{k-1}) + \Phi^n_{k/k-1}\delta X + \frac{1}{2}\sum_{i=1}^n e_i \mathrm{tr}\left(D_{fi}\cdot\delta X\delta X^{\mathrm{T}}\right) + \Gamma_{k-1}W_{k-1}$$

状态一步预测

$$\hat{X}_{k/k-1} = \mathrm{E}\left[f(X^n_{k-1}) + \Phi^n_{k/k-1}\delta X + \frac{1}{2}\sum_{i=1}^n e_i \mathrm{tr}\left(D_{fi}\cdot\delta X\delta X^{\mathrm{T}}\right) + \Gamma_{k-1}W_{k-1}\right]$$

$$= f(X^n_{k-1}) + \Phi^n_{k/k-1}\mathrm{E}[\delta X] + \frac{1}{2}\sum_{i=1}^n e_i \mathrm{tr}\left(D_{fi}\cdot \mathrm{E}[\delta X\delta X^{\mathrm{T}}]\right) + \Gamma_{k-1}\mathrm{E}[W_{k-1}]$$

0均值　　0均值

$$= f(X^n_{k-1}) + \frac{1}{2}\sum_{i=1}^n e_i \mathrm{tr}\left(D_{fi}P_{k-1}\right)$$

→体现非线性

状态一步预测误差

$$\tilde{X}_{k/k-1} = X_k - \hat{X}_{k/k-1}$$

$$= \Phi^n_{k/k-1}\delta X + \frac{1}{2}\sum_{i=1}^n e_i \mathrm{tr}\left(D_{fi}\cdot\delta X\delta X^{\mathrm{T}}\right) + \Gamma_{k-1}W_{k-1} - \frac{1}{2}\sum_{i=1}^n e_i \mathrm{tr}\left(D_{fi}P_{k-1}\right)$$

$$= \Phi^n_{k/k-1}\delta X + \frac{1}{2}\sum_{i=1}^n e_i \mathrm{tr}\left(D_{fi}(\delta X\delta X^{\mathrm{T}} - P_{k-1})\right) + \Gamma_{k-1}W_{k-1}$$

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$$\tilde{\boldsymbol{X}}_{k/k-1}=\boldsymbol{\Phi}_{k/k-1}^{n}\delta\boldsymbol{X}+\frac{1}{2}\sum_{i=1}^{n}\boldsymbol{e}_i\operatorname{tr}\left(\boldsymbol{D}_{fi}(\delta\boldsymbol{X}\delta\boldsymbol{X}^{\mathrm{T}}-\boldsymbol{P}_{k-1})\right)+\boldsymbol{\Gamma}_{k-1}\boldsymbol{W}_{k-1}$$

状态一步预测误差方差阵

$$\begin{aligned}
\boldsymbol{P}_{k/k-1}&=\mathrm{E}[\tilde{\boldsymbol{X}}_{k/k-1}\tilde{\boldsymbol{X}}_{k/k-1}^{\mathrm{T}}]\\
&=\mathrm{E}\Bigg[\left[\boldsymbol{\Phi}_{k/k-1}^{n}\delta\boldsymbol{X}+\frac{1}{2}\sum_{i=1}^{n}\boldsymbol{e}_i\operatorname{tr}\left(\boldsymbol{D}_{fi}(\delta\boldsymbol{X}\delta\boldsymbol{X}^{\mathrm{T}}-\boldsymbol{P}_{k-1})\right)+\boldsymbol{\Gamma}_{k-1}\boldsymbol{W}_{k-1}\right]\\
&\qquad\times\left[\boldsymbol{\Phi}_{k/k-1}^{n}\delta\boldsymbol{X}+\frac{1}{2}\sum_{i=1}^{n}\boldsymbol{e}_i\operatorname{tr}\left(\boldsymbol{D}_{fi}(\delta\boldsymbol{X}\delta\boldsymbol{X}^{\mathrm{T}}-\boldsymbol{P}_{k-1})\right)+\boldsymbol{\Gamma}_{k-1}\boldsymbol{W}_{k-1}\right]^{\mathrm{T}}\Bigg]\\
&=\boldsymbol{\Phi}_{k/k-1}^{n}\boldsymbol{P}_{k-1}(\boldsymbol{\Phi}_{k/k-1}^{n})^{\mathrm{T}}\\
&\quad+\frac{1}{4}\mathrm{E}\left[\sum_{i=1}^{n}\boldsymbol{e}_i\operatorname{tr}\left(\boldsymbol{D}_{fi}(\delta\boldsymbol{X}\delta\boldsymbol{X}^{\mathrm{T}}-\boldsymbol{P}_{k-1})\right)\left(\sum_{i=1}^{n}\boldsymbol{e}_i\operatorname{tr}\left(\boldsymbol{D}_{fi}(\delta\boldsymbol{X}\delta\boldsymbol{X}^{\mathrm{T}}-\boldsymbol{P}_{k-1})\right)\right)^{\mathrm{T}}\right]+\boldsymbol{\Gamma}_{k-1}\boldsymbol{Q}_{k-1}\boldsymbol{\Gamma}_{k-1}^{\mathrm{T}}\\
&=\boldsymbol{\Phi}_{k/k-1}^{n}\boldsymbol{P}_{k-1}(\boldsymbol{\Phi}_{k/k-1}^{n})^{\mathrm{T}}\\
&\quad+\frac{1}{4}\sum_{i=1}^{n}\sum_{j=1}^{n}\boldsymbol{e}_i\boldsymbol{e}_j^{\mathrm{T}}\mathrm{E}\left[\operatorname{tr}\left(\boldsymbol{D}_{fi}(\delta\boldsymbol{X}\delta\boldsymbol{X}^{\mathrm{T}}-\boldsymbol{P}_{k-1})\right)\cdot\operatorname{tr}\left(\boldsymbol{D}_{fj}(\delta\boldsymbol{X}\delta\boldsymbol{X}^{\mathrm{T}}-\boldsymbol{P}_{k-1})\right)\right]+\boldsymbol{\Gamma}_{k-1}\boldsymbol{Q}_{k-1}\boldsymbol{\Gamma}_{k-1}^{\mathrm{T}}\\
&=\boldsymbol{\Phi}_{k/k-1}^{n}\boldsymbol{P}_{k-1}(\boldsymbol{\Phi}_{k/k-1}^{n})^{\mathrm{T}}+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\boldsymbol{e}_i\boldsymbol{e}_j^{\mathrm{T}}\operatorname{tr}\left(\boldsymbol{D}_{fi}\boldsymbol{P}_{k-1}\boldsymbol{D}_{fj}\boldsymbol{P}_{k-1}\right)+\boldsymbol{\Gamma}_{k-1}\boldsymbol{Q}_{k-1}\boldsymbol{\Gamma}_{k-1}^{\mathrm{T}}
\end{aligned}$$

↳ 四阶矩

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验证（上页）：

$$\begin{aligned}
&\mathrm{E}\left[\operatorname{tr}\left(\boldsymbol{D}_{fi}(\delta\boldsymbol{X}\delta\boldsymbol{X}^{\mathrm{T}}-\boldsymbol{P}_{k-1})\right)\cdot\operatorname{tr}\left(\boldsymbol{D}_{fj}(\delta\boldsymbol{X}\delta\boldsymbol{X}^{\mathrm{T}}-\boldsymbol{P}_{k-1})\right)\right]\\
&\quad=\mathrm{E}\Big[\operatorname{tr}(\boldsymbol{D}_{fi}\cdot\delta\boldsymbol{X}\delta\boldsymbol{X}^{\mathrm{T}})\cdot\operatorname{tr}(\boldsymbol{D}_{fj}\cdot\delta\boldsymbol{X}\delta\boldsymbol{X}^{\mathrm{T}})-\operatorname{tr}(\boldsymbol{D}_{fi}\cdot\delta\boldsymbol{X}\delta\boldsymbol{X}^{\mathrm{T}})\cdot\operatorname{tr}(\boldsymbol{D}_{fj}\boldsymbol{P}_{k-1})\\
&\qquad-\operatorname{tr}(\boldsymbol{D}_{fi}\boldsymbol{P}_{k-1})\cdot\operatorname{tr}(\boldsymbol{D}_{fj}\cdot\delta\boldsymbol{X}\delta\boldsymbol{X}^{\mathrm{T}})+\operatorname{tr}(\boldsymbol{D}_{fi}\boldsymbol{P}_{k-1})\cdot\operatorname{tr}(\boldsymbol{D}_{fj}\boldsymbol{P}_{k-1})\Big]\\
&\quad=\mathrm{E}\left[\operatorname{tr}(\boldsymbol{D}_{fi}\cdot\delta\boldsymbol{X}\delta\boldsymbol{X}^{\mathrm{T}})\cdot\operatorname{tr}(\boldsymbol{D}_{fj}\cdot\delta\boldsymbol{X}\delta\boldsymbol{X}^{\mathrm{T}})\right]-\operatorname{tr}(\boldsymbol{D}_{fi}\boldsymbol{P}_{k-1})\cdot\operatorname{tr}(\boldsymbol{D}_{fj}\boldsymbol{P}_{k-1})\\
&\quad=2\operatorname{tr}(\boldsymbol{D}_{fi}\boldsymbol{P}_{k-1}\boldsymbol{D}_{fj}\boldsymbol{P}_{k-1})
\end{aligned}$$

→ 用记着吧

$$\mathrm{E}\left[\operatorname{tr}(\boldsymbol{A}\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}})\operatorname{tr}(\boldsymbol{B}\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}})\right]=\sum_{l=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}A_{ij}B_{kl}\mathrm{E}[X_iX_jX_kX_l]$$

$$\operatorname{tr}(\boldsymbol{APBP})=\sum_{l=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}A_{ij}B_{kl}P_{il}P_{jk}$$

$$\operatorname{tr}(\boldsymbol{AP})\operatorname{tr}(\boldsymbol{BP})=\sum_{l=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}A_{ij}B_{kl}P_{ij}P_{kl}$$

四阶混合矩计算公式

$$\mathrm{E}[X_iX_jX_kX_l]=P_{ij}P_{kl}+P_{ik}P_{jl}+P_{il}P_{jk}$$

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量测方程的二阶泰勒级数展开

$$\boldsymbol{Z}_{k/k-1}=\boldsymbol{h}(\boldsymbol{X}_{k/k-1}^{n})+\boldsymbol{H}_{k}^{n}\delta\boldsymbol{X}+\frac{1}{2}\sum_{i=1}^{m}\boldsymbol{e}_{i}\mathrm{tr}\left(\boldsymbol{D}_{hi}\cdot\delta\boldsymbol{X}\delta\boldsymbol{X}^{\mathrm{T}}\right)+\boldsymbol{V}_{k}$$

量测一步预测

$$\hat{\boldsymbol{Z}}_{k/k-1}=\boldsymbol{h}(\boldsymbol{X}_{k/k-1}^{n})+\frac{1}{2}\sum_{i=1}^{m}\boldsymbol{e}_{i}\mathrm{tr}\left(\boldsymbol{D}_{hi}\boldsymbol{P}_{k/k-1}\right)$$

量测一步预测方差阵、状态/量测协方差阵

$$\boldsymbol{P}_{ZZ,k/k-1}=\boldsymbol{H}_{k}^{n}\boldsymbol{P}_{k/k-1}(\boldsymbol{H}_{k}^{n})^{\mathrm{T}}+\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\boldsymbol{e}_{i}\boldsymbol{e}_{j}^{\mathrm{T}}\mathrm{tr}\left(\boldsymbol{D}_{hi}\boldsymbol{P}_{k/k-1}\boldsymbol{D}_{hj}\boldsymbol{P}_{k/k-1}\right)+\boldsymbol{R}_{k}$$

$$\boldsymbol{P}_{XZ,k/k-1}=\boldsymbol{P}_{k/k-1}\boldsymbol{H}_{k}^{n}$$

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最后，得二阶滤波公式

运算量巨大而收益微小！

$$\begin{cases}\hat{\boldsymbol{X}}_{k/k-1}=\boldsymbol{f}(\hat{\boldsymbol{X}}_{k-1})+\frac{1}{2}\sum_{i=1}^{n}\boldsymbol{e}_{i}\mathrm{tr}\left(\boldsymbol{D}_{fi}\boldsymbol{P}_{k-1}\right)\\ \boldsymbol{P}_{k/k-1}=\boldsymbol{\Phi}_{k/k-1}\boldsymbol{P}_{k-1}\boldsymbol{\Phi}_{k/k-1}^{\mathrm{T}}+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\boldsymbol{e}_{i}\boldsymbol{e}_{j}^{\mathrm{T}}\mathrm{tr}\left(\boldsymbol{D}_{fi}\boldsymbol{P}_{k-1}\boldsymbol{D}_{fj}\boldsymbol{P}_{k-1}\right)+\boldsymbol{\Gamma}_{k-1}\boldsymbol{Q}_{k-1}\boldsymbol{\Gamma}_{k-1}^{\mathrm{T}}\\ \boldsymbol{K}_{k}=\boldsymbol{P}_{k/k-1}\boldsymbol{H}_{k}^{\mathrm{T}}\left[\boldsymbol{H}_{k}\boldsymbol{P}_{k/k-1}\boldsymbol{H}_{k}^{\mathrm{T}}+\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\boldsymbol{e}_{i}\boldsymbol{e}_{j}^{\mathrm{T}}\mathrm{tr}(\boldsymbol{D}_{hi}\boldsymbol{P}_{k/k-1}\boldsymbol{D}_{hj}\boldsymbol{P}_{k/k-1})+\boldsymbol{R}_{k}\right]^{-1}\\ \hat{\boldsymbol{X}}_{k}=\hat{\boldsymbol{X}}_{k/k-1}+\boldsymbol{K}_{k}\left[\boldsymbol{Z}_{k}-\boldsymbol{h}(\hat{\boldsymbol{X}}_{k/k-1})-\frac{1}{2}\sum_{i=1}^{m}\boldsymbol{e}_{i}\mathrm{tr}\left(\boldsymbol{D}_{hi}\boldsymbol{P}_{k/k-1}\right)\right]\\ \boldsymbol{P}_{k}=(\boldsymbol{I}-\boldsymbol{K}_{k}\boldsymbol{H}_{k})\boldsymbol{P}_{k/k-1}\end{cases}$$

→运算量：$n^2\cdot3$个 n阶矩阵乘法、

实际没用！

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13.4 迭代滤波

基本思想

$\hat{X}_{k-1/k}$ →反向平滑

EKF
$$\begin{cases} \hat{X}_{k/k-1} = f(\hat{X}_{k-1}) \\ P_{k/k-1} = \Phi_{k/k-1} P_{k-1} \Phi_{k/k-1}^{\mathrm{T}} + \Gamma_{k-1} Q_{k-1} \Gamma_{k-1}^{\mathrm{T}} \\ K_k = P_{k/k-1} H_k^{\mathrm{T}} (H_k P_{k/k-1} H_k^{\mathrm{T}} + R_k)^{-1} \\ \hat{X}_k = \hat{X}_{k/k-1} + K_k [Z_k - h(\hat{X}_{k/k-1})] \\ P_k = (I - K_k H_k) P_{k/k-1} \end{cases}$$

$$\Phi_{k/k-1} = J(f(\hat{X}_{k-1}))$$
$$H_k = J(h(\hat{X}_{k/k-1}))$$

思路：越接近展开点，精度越高
↓
先滤一次，再反向平滑
↓
再滤。

（1）预滤波

$$\begin{cases} \hat{X}_{k/k-1}^* = f(\hat{X}_{k-1}) \\ P_{k/k-1}^* = \Phi_{k/k-1}^* P_{k-1} (\Phi_{k/k-1}^*)^{\mathrm{T}} + \Gamma_{k-1} Q_{k-1} \Gamma_{k-1}^{\mathrm{T}} \\ K_k^* = P_{k/k-1}^* (H_k^*)^{\mathrm{T}} [H_k^* P_{k/k-1}^* (H_k^*)^{\mathrm{T}} + R_k]^{-1} \\ \hat{X}_k^* = \hat{X}_{k/k-1}^* + K_k^* [Z_k - h(\hat{X}_{k/k-1}^*)] \end{cases}$$

$$\Phi_{k/k-1}^* = J(f(\hat{X}_{k-1}))$$
$$H_k^* = J(h(\hat{X}_{k/k-1}^*))$$

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$$\begin{cases} K_{s,k} = P_{f,k} \Phi_{k+1/k}^{\mathrm{T}} P_{f,k+1/k}^{-1} \\ \hat{X}_{s,k} = \hat{X}_{f,k} + K_{s,k} (\hat{X}_{s,k+1} - \hat{X}_{f,k+1/k}) \\ P_{s,k} = P_{f,k} + K_{s,k} (P_{s,k+1} - P_{f,k+1/k}) K_{s,k}^{\mathrm{T}} \end{cases}$$

（2）反向一步平滑

$$\hat{X}_{k-1/k} = \hat{X}_{k-1} + P_{k-1} (\Phi_{k/k-1}^*)^{\mathrm{T}} (P_{k/k-1}^*)^{-1} (\hat{X}_k^* - \hat{X}_{k/k-1}^*)$$

重做状态一步预测
$$\hat{X}_{k/k-1} = f(\hat{X}_{k-1}) = f(\hat{X}_{k-1/k}) + f(\hat{X}_{k-1}) - f(\hat{X}_{k-1/k}) \approx f(\hat{X}_{k-1/k}) + J\left(f(\hat{X}_{k-1/k})\right)(\hat{X}_{k-1} - \hat{X}_{k-1/k})$$

重做量测一步预测
$$\hat{Z}_{k/k-1} = h(\hat{X}_{k/k-1}) = h(\hat{X}_k^*) + h(\hat{X}_{k/k-1}) - h(\hat{X}_k^*) \approx h(\hat{X}_k^*) + J\left(h(\hat{X}_k^*)\right)(\hat{X}_{k/k-1} - \hat{X}_k^*)$$

（3）迭代滤波

$$\begin{cases} \hat{X}_{k/k-1} = f(\hat{X}_{k-1/k}) + \Phi_{k/k-1} (\hat{X}_{k-1} - \hat{X}_{k-1/k}) \\ P_{k/k-1} = \Phi_{k/k-1} P_{k-1} \Phi_{k/k-1}^{\mathrm{T}} + \Gamma_{k-1} Q_{k-1} \Gamma_{k-1}^{\mathrm{T}} \\ K_k = P_{k/k-1} H_k^{\mathrm{T}} (H_k P_{k/k-1} H_k^{\mathrm{T}} + R_k)^{-1} \\ \hat{X}_k = \hat{X}_{k/k-1} + K_k [Z_k - h(\hat{X}_k^*) - H_k (\hat{X}_{k/k-1} - \hat{X}_k^*)] \\ P_k = (I - K_k H_k) P_{k/k-1} \end{cases}$$

$$\Phi_{k/k-1} = J(f(\hat{X}_{k-1/k}))$$
$$H_k = J(h(\hat{X}_k^*))$$

还可多次迭代，但收益小！ 一次就够了！

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<注>1、非线性与离散化间隔有关，用二阶滤波不如把间隔变短，用一阶。

2、对于稳定系统，高阶算法可能比较好！

2、对于确定系统，高阶算法可能比较好！
对于随机系统，用低阶算法高频率进行（相对于间隔变小）

3、应用中，建模建成连续性系统 ⇒ 离散化 ⇒ 线性化，结果一样
线性化 ⇒ 离散化