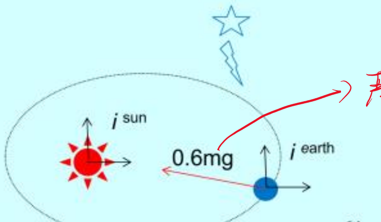
# 捷联惯导数值更新算法

2023年1月16日 20:42

## 3 捷联惯导更新算法

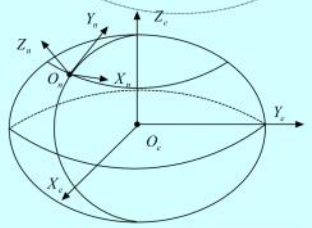
常用坐标系定义：

(1)惯性坐标系($i$系，日心惯性/地心准惯性坐标系)




严格来说，地心系不是惯性系(绕太阳转)
但导航目标在地球上，惯系为地球，所以0.6mg没影响

$\boldsymbol{\omega}_{ie}$



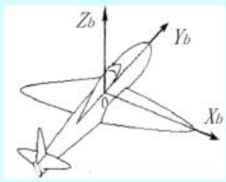

此节 (2)地球坐标系($e$系，ECEF)
(3)导航坐标系($n$系，E-N-U)
东-北-天

$\boldsymbol{\omega}_{en}$

$\boldsymbol{C}_n^e(L,\lambda)$

$\boldsymbol{C}_b^n(\theta,\gamma,\psi)$

(4)载体坐标系($b$系，R-F-U)

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## 3 捷联惯导更新算法

没法测出，未知
就消相当于惯系的速度可用陀螺测出，

### 3.1 姿态更新

$\boldsymbol{\omega}_{ib}=\boldsymbol{\omega}_{iX}+\boldsymbol{\omega}_{Xb}$ → $i \to n \to b$

由姿态阵微分方程 $\dot{\boldsymbol{C}}_b^i=\boldsymbol{C}_b^i(\boldsymbol{\omega}_{ib}^b\times)$ 可得(符号替换i->n)：$b$系相对$n$系

$$\dot{\boldsymbol{C}}_b^n=\boldsymbol{C}_b^n(\boldsymbol{\omega}_{nb}^b\times)=\boldsymbol{C}_b^n\left[(\boldsymbol{\omega}_{ib}^b-\boldsymbol{\omega}_{in}^b)\times\right]=\boldsymbol{C}_b^n(\boldsymbol{\omega}_{ib}^b\times)-\boldsymbol{C}_b^n(\boldsymbol{\omega}_{in}^b\times)$$

$$=\boldsymbol{C}_b^n(\boldsymbol{\omega}_{ib}^b\times)-\boxed{\boldsymbol{C}_b^n(\boldsymbol{\omega}_{in}^b\times)\boldsymbol{C}_n^b}\boldsymbol{C}_b^n=\boldsymbol{C}_b^n(\boldsymbol{\omega}_{ib}^b\times)-(\boldsymbol{\omega}_{in}^n\times)\boldsymbol{C}_b^n$$

测量量　计算量 $\boldsymbol{\omega}_{ie}+\boldsymbol{\omega}_{en}$

实际不用该思路！

直接采用姿态阵链乘拆解方法

$$\boldsymbol{C}_{b(m)}^{n(m)}=\boldsymbol{C}_i^{n(m)}\boldsymbol{C}_{b(m)}^i=\boldsymbol{C}_{n(m-1)}^{n(m)}\boldsymbol{C}_i^{n(m-1)}\boldsymbol{C}_{b(m-1)}^i\boldsymbol{C}_{b(m)}^{b(m-1)}$$

m,m-1为时刻

$$=\boldsymbol{C}_{n(m-1)}^{n(m)}\boldsymbol{C}_{b(m-1)}^{n(m-1)}\boldsymbol{C}_{b(m)}^{b(m-1)}$$

上面讲过的姿态角算

其中

$$\boldsymbol{C}_{b(m)}^{b(m-1)}=\boldsymbol{M}_{RV}(\boldsymbol{\phi}_{ib(m)}^b)\qquad \boldsymbol{\phi}_{ib(m)}^b=(\Delta\boldsymbol{\theta}_{m1}+\Delta\boldsymbol{\theta}_{m2})+\frac{2}{3}\Delta\boldsymbol{\theta}_{m1}\times\Delta\boldsymbol{\theta}_{m2}$$

$$\boldsymbol{C}_{n(m-1)}^{n(m)}=(\boldsymbol{C}_{n(m)}^{n(m-1)})^{\mathrm{T}}=\boldsymbol{M}_{RV}^{\mathrm{T}}(\boldsymbol{\phi}_{in(m)}^n)\qquad \boldsymbol{\phi}_{in(m)}^n\approx T_m\boldsymbol{\omega}_{in}^n$$

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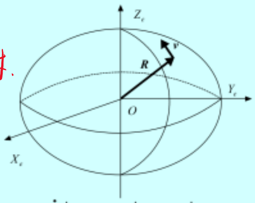
导航系相对于地球系

F=ma

m=1时

F=a

## 3 捷联惯导更新算法




具体原理,推导见书

### 3.2 速度更新—（1）比力方程

$$\boldsymbol{R}_{eg}^{e}=\boldsymbol{C}_{i}^{e}\boldsymbol{R}_{eg}^{i}$$

$$\dot{\boldsymbol{R}}_{eg}^{e}=\boldsymbol{C}_{i}^{e}\dot{\boldsymbol{R}}_{eg}^{i}+\dot{\boldsymbol{C}}_{i}^{e}\boldsymbol{R}_{eg}^{i}=\boldsymbol{C}_{i}^{e}\dot{\boldsymbol{R}}_{eg}^{i}+\boldsymbol{C}_{i}^{e}(\boldsymbol{\omega}_{ei}^{i}\times)\boldsymbol{R}_{eg}^{i}=\boldsymbol{C}_{i}^{e}[\dot{\boldsymbol{R}}_{eg}^{i}-(\boldsymbol{\omega}_{ie}^{i}\times)\boldsymbol{R}_{eg}^{i}]$$

$$\boldsymbol{C}_{e}^{g}\dot{\boldsymbol{R}}_{eg}^{e}=\boldsymbol{C}_{e}^{g}\boldsymbol{C}_{i}^{e}[\dot{\boldsymbol{R}}_{eg}^{i}-(\boldsymbol{\omega}_{ie}^{i}\times)\boldsymbol{R}_{eg}^{i}]=\boldsymbol{C}_{i}^{g}[\dot{\boldsymbol{R}}_{eg}^{i}-(\boldsymbol{\omega}_{ie}^{i}\times)\boldsymbol{R}_{eg}^{i}]$$

$$\boldsymbol{v}_{eg}^{g}=\boldsymbol{C}_{e}^{g}\dot{\boldsymbol{R}}_{eg}^{e}=\boldsymbol{C}_{i}^{g}[\dot{\boldsymbol{R}}_{eg}^{i}-(\boldsymbol{\omega}_{ie}^{i}\times)\boldsymbol{R}_{eg}^{i}]$$

$$\dot{\boldsymbol{v}}_{eg}^{g}=\dot{\boldsymbol{C}}_{i}^{g}[\dot{\boldsymbol{R}}_{eg}^{i}-(\boldsymbol{\omega}_{ie}^{i}\times)\boldsymbol{R}_{eg}^{i}]+\boldsymbol{C}_{i}^{g}[\ddot{\boldsymbol{R}}_{eg}^{i}-(\boldsymbol{\omega}_{ie}^{i}\times)\dot{\boldsymbol{R}}_{eg}^{i}]$$

$$=\boldsymbol{C}_{i}^{g}(\boldsymbol{\omega}_{gi}^{i}\times)[\dot{\boldsymbol{R}}_{eg}^{i}-(\boldsymbol{\omega}_{ie}^{i}\times)\boldsymbol{R}_{eg}^{i}]+\boldsymbol{C}_{i}^{g}[\ddot{\boldsymbol{R}}_{eg}^{i}-(\boldsymbol{\omega}_{ie}^{i}\times)\dot{\boldsymbol{R}}_{eg}^{i}]$$

$$\dot{\boldsymbol{R}}_{eg}^{i}-(\boldsymbol{\omega}_{ie}^{i}\times)\boldsymbol{R}_{eg}^{i}=\boldsymbol{C}_{g}^{i}\boldsymbol{v}_{eg}^{g}$$

$$\dot{\boldsymbol{R}}_{eg}^{i}=\boldsymbol{C}_{g}^{i}\boldsymbol{v}_{eg}^{g}+(\boldsymbol{\omega}_{ie}^{i}\times)\boldsymbol{R}_{eg}^{i}$$

$$\dot{\boldsymbol{v}}_{eg}^{g}=\boldsymbol{C}_{i}^{g}(\boldsymbol{\omega}_{gi}^{i}\times)\boldsymbol{C}_{g}^{i}\boldsymbol{v}_{eg}^{g}+\boldsymbol{C}_{i}^{g}\{\ddot{\boldsymbol{R}}_{eg}^{i}-(\boldsymbol{\omega}_{ie}^{i}\times)[\boldsymbol{C}_{g}^{i}\boldsymbol{v}_{eg}^{g}+(\boldsymbol{\omega}_{ie}^{i}\times)\boldsymbol{R}_{eg}^{i}]\}$$

$$=\boldsymbol{C}_{i}^{g}[\ddot{\boldsymbol{R}}_{eg}^{i}-(\boldsymbol{\omega}_{ie}^{i}\times)^{2}\boldsymbol{R}_{eg}^{i}]-\boldsymbol{C}_{i}^{g}[(\boldsymbol{\omega}_{ie}^{i}\times)-(\boldsymbol{\omega}_{gi}^{i}\times)]\boldsymbol{C}_{g}^{i}\boldsymbol{v}_{eg}^{g}$$

$$\ddot{\boldsymbol{R}}_{eg}^{i}=\ddot{\boldsymbol{R}}_{ig}^{i}=\boldsymbol{f}_{sf}^{i}+\boldsymbol{G}^{i}$$

$$=\boldsymbol{C}_{i}^{g}[\ddot{\boldsymbol{R}}_{eg}^{i}-(\boldsymbol{\omega}_{ie}^{i}\times)^{2}\boldsymbol{R}_{eg}^{i}]-[(\boldsymbol{\omega}_{ie}^{g}\times)+(\boldsymbol{\omega}_{ig}^{g}\times)]\boldsymbol{v}_{eg}^{g}$$

$$\boldsymbol{g}^{i}=\boldsymbol{G}^{i}-(\boldsymbol{\omega}_{ie}^{i}\times)^{2}\boldsymbol{R}_{ig}^{i}=\boldsymbol{G}^{i}-(\boldsymbol{\omega}_{ie}^{i}\times)^{2}\boldsymbol{R}_{eg}^{i}$$

$$\dot{\boldsymbol{v}}_{eg}^{g}=\boldsymbol{C}_{i}^{g}[\boldsymbol{f}_{sf}^{i}+\boldsymbol{G}^{i}-(\boldsymbol{\omega}_{ie}^{i}\times)^{2}\boldsymbol{R}_{eg}^{i}]-[(\boldsymbol{\omega}_{ie}^{g}\times)+(\boldsymbol{\omega}_{ig}^{g}\times)]\boldsymbol{v}_{eg}^{g}=\boldsymbol{C}_{i}^{g}(\boldsymbol{f}_{sf}^{i}+\boldsymbol{g}^{i})-[(\boldsymbol{\omega}_{ie}^{g}\times)+(\boldsymbol{\omega}_{ig}^{g}\times)]\boldsymbol{v}_{eg}^{g}$$

$$=\boldsymbol{f}_{sf}^{g}+\boldsymbol{g}^{g}-[2(\boldsymbol{\omega}_{ie}^{g}\times)+(\boldsymbol{\omega}_{eg}^{g}\times)]\boldsymbol{v}_{eg}^{g}=\boldsymbol{C}_{b}^{g}\boldsymbol{f}_{sf}^{b}-[2(\boldsymbol{\omega}_{ie}^{g}\times)+(\boldsymbol{\omega}_{eg}^{g}\times)]\boldsymbol{v}_{eg}^{g}+\boldsymbol{g}^{g}$$

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类似于惯性系下的F=ma　　哥氏加速度　　圆周运动修正　　重力修正

## 3 捷联惯导更新算法

$$\dot{\boldsymbol{v}}^{n}=\boldsymbol{C}_{b}^{n}\boldsymbol{f}_{sf}^{b}-[2(\boldsymbol{\omega}_{ie}^{n}\times)+(\boldsymbol{\omega}_{en}^{n}\times)]\boldsymbol{v}^{n}+\boldsymbol{g}^{n}$$

（2）比力方程的离散化更新方法

$$\dot{\boldsymbol{v}}^{n}(t)=\boldsymbol{C}_{b}^{n}(t)\boldsymbol{f}_{sf}^{b}(t)-\{2[\boldsymbol{\omega}_{ie}^{n}(t)\times]+[\boldsymbol{\omega}_{en}^{n}(t)\times]\}\boldsymbol{v}^{n}(t)+\boldsymbol{g}^{n}(t)$$

上式积分，记为

角运动　线运动

$$\boldsymbol{v}_{m}^{n(m)}-\boldsymbol{v}_{m-1}^{n(m-1)}=\int_{t_{m-1}}^{t_m}\boldsymbol{C}_{b}^{n}(t)\boldsymbol{f}_{sf}^{b}(t)\mathrm{d}t+\int_{t_{m-1}}^{t_m}-\{2[\boldsymbol{\omega}_{ie}^{n}(t)\times]+[\boldsymbol{\omega}_{en}^{n}(t)\times]\}\boldsymbol{v}^{n}(t)+\boldsymbol{g}^{n}(t)\mathrm{d}t$$

$$=\Delta\boldsymbol{v}_{sf(m)}^{n}+\Delta\boldsymbol{v}_{cor/g(m)}^{n}$$

$$\boldsymbol{v}_{m}^{n(m)}=\boldsymbol{v}_{m-1}^{n(m-1)}+\Delta\boldsymbol{v}_{sf(m)}^{n}+\Delta\boldsymbol{v}_{cor/g(m)}^{n}$$

其中，快变部分

前一刻的姿态阵,变化较小

$$\boldsymbol{C}=\boldsymbol{I}+\frac{\sin\phi}{\phi}(\boldsymbol{\phi}\times)+\frac{1-\cos\phi}{\phi^{2}}(\boldsymbol{\phi}\times)^{2}$$

$$\approx\boldsymbol{I}+(\boldsymbol{\phi}\times)$$

$$\Delta\boldsymbol{v}_{sf(m)}^{n}=\int_{t_{m-1}}^{t_m}\boldsymbol{C}_{b}^{n}(t)\boldsymbol{f}_{sf}^{b}(t)\mathrm{d}t=\int_{t_{m-1}}^{t_m}\boldsymbol{C}_{n(m-1)}^{n(t)}\boldsymbol{C}_{b(m-1)}^{n(m-1)}\boldsymbol{C}_{b(t)}^{b(m-1)}\boldsymbol{f}_{sf}^{b}(t)\mathrm{d}t$$

$$\approx\int_{t_{m-1}}^{t_m}[\boldsymbol{I}-\boldsymbol{\phi}_{in}^{n}(t,t_{m-1})\times]\boldsymbol{C}_{b(m-1)}^{n(m-1)}[\boldsymbol{I}+\boldsymbol{\phi}_{ib}^{b}(t,t_{m-1})\times]\boldsymbol{f}_{sf}^{b}(t)\mathrm{d}t$$

$$\approx\int_{t_{m-1}}^{t_m}\boldsymbol{C}_{b(m-1)}^{n(m-1)}\boldsymbol{f}_{sf}^{b}(t)-[\boldsymbol{\phi}_{in}^{n}(t,t_{m-1})\times]\boldsymbol{C}_{b(m-1)}^{n(m-1)}\boldsymbol{f}_{sf}^{b}(t)+\boldsymbol{C}_{b(m-1)}^{n(m-1)}[\boldsymbol{\phi}_{ib}^{b}(t,t_{m-1})\times]\boldsymbol{f}_{sf}^{b}(t)\mathrm{d}t$$

比力增量

$$=\boldsymbol{C}_{b(m-1)}^{n(m-1)}\Delta\boldsymbol{v}_{sf(m)}^{b}-\int_{t_{m-1}}^{t_m}[\boldsymbol{\phi}_{in}^{n}(t,t_{m-1})\times]\boldsymbol{C}_{b(m-1)}^{n(m-1)}\boldsymbol{f}_{sf}^{b}(t)\mathrm{d}t+\boldsymbol{C}_{b(m-1)}^{n(m-1)}\int_{t_{m-1}}^{t_m}\boldsymbol{\phi}_{ib}^{b}(t,t_{m-1})\times\boldsymbol{f}_{sf}^{b}(t)\mathrm{d}t$$

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常值　加速度计采样获得

①　②　③

## 3 捷联惯导更新算法

$$\Delta\boldsymbol{v}_{sf(m)}^{n}=\boldsymbol{C}_{b(m-1)}^{n(m-1)}\Delta\boldsymbol{v}_{sf(m)}^{b}-\int_{t_{m-1}}^{t_m}[\boldsymbol{\phi}_{in}^{n}(t,t_{m-1})\times]\boldsymbol{C}_{b(m-1)}^{n(m-1)}\boldsymbol{f}_{sf}^{b}(t)\mathrm{d}t+\boldsymbol{C}_{b(m-1)}^{n(m-1)}\int_{t_{m-1}}^{t_m}\boldsymbol{\phi}_{ib}^{b}(t,t_{m-1})\times\boldsymbol{f}_{sf}^{b}(t)\mathrm{d}t$$

作近似 $\boldsymbol{\phi}_{in}^{n}(t,t_{m-1})\approx(t-t_{m-1})\boldsymbol{\omega}_{in}^{n}(t_{m-1/2})$，则有　微小旋转矢量近似成角速度

$$\int_{t_{m-1}}^{t_m}[\boldsymbol{\phi}_{in}^{n}(t,t_{m-1})\times]\boldsymbol{C}_{b(m-1)}^{n(m-1)}\boldsymbol{f}_{sf}^{b}(t)\mathrm{d}t\approx\int_{t_{m-1}}^{t_m}\left[(t-t_{m-1})\boldsymbol{\omega}_{in}^{n}(t_{m-1/2})\times\right]\boldsymbol{C}_{b(m-1)}^{n(m-1)}\boldsymbol{f}_{sf}^{b}(t)\mathrm{d}t$$

$$=[\boldsymbol{\omega}_{in}^{n}(t_{m-1/2})\times]\boldsymbol{C}_{b(m-1)}^{n(m-1)}\int_{t_{m-1}}^{t_m}(t-t_{m-1})\boldsymbol{f}_{sf}^{b}(t)\mathrm{d}t$$

$$\approx[\boldsymbol{\omega}_{in}^{n}(t_{m-1/2})\times]\boldsymbol{C}_{b(m-1)}^{n(m-1)}\int_{t_{m-1}}^{t_m}(t-t_{m-1})\boldsymbol{f}_{sf}^{b}(t_{m-1/2})\mathrm{d}(t-t_{m-1})$$

$$\approx[\boldsymbol{\omega}_{in}^{n}(t_{m-1/2})\times]\boldsymbol{C}_{b(m-1)}^{n(m-1)}\frac{1}{2}(t_m-t_{m-1})^{2}\boldsymbol{f}_{sf}^{b}(t_{m-1/2})$$

$$=\frac{T_m}{2}[\boldsymbol{\omega}_{in}^{n}(t_{m-1/2})\times]\boldsymbol{C}_{b(m-1)}^{n(m-1)}\boxed{\boldsymbol{f}_{sf}^{b}(t_{m-1/2})T_m}$$

$$=\frac{T_m}{2}[\boldsymbol{\omega}_{in}^{n}(t_{m-1/2})\times]\boldsymbol{C}_{b(m-1)}^{n(m-1)}\boxed{\Delta\boldsymbol{v}_{sf}^{b}(t_m)}$$

导航系旋转修正量

短时间内 <<1

$$= \frac{T_m}{2}[\boldsymbol{\omega}_{in}^n(t_{m-1/2})\times]\boldsymbol{C}_{b(m-1)}^{n(m-1)}\Delta\boldsymbol{v}_{sf}^b(t_m)$$ 导航系旋转修正量

短时间内 <<1

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$$\Delta\boldsymbol{v}_{sf(m)}^n = \boldsymbol{C}_{b(m-1)}^{n(m-1)}\Delta\boldsymbol{v}_{sf(m)}^b - \int_{t_{m-1}}^{t_m}[\boldsymbol{\phi}_{in}^n(t,t_{m-1})\times]\boldsymbol{C}_{b(m-1)}^{n(m-1)}\boldsymbol{f}_{sf}^b(t)\,\mathrm{d}t + \boldsymbol{C}_{b(m-1)}^{n(m-1)}\int_{t_{m-1}}^{t_m}\boldsymbol{\phi}_{ib}^b(t,t_{m-1})\times\boldsymbol{f}_{sf}^b(t)\,\mathrm{d}t$$

## 3 捷联惯导更新算法

作近似 $\int_{t_{m-1}}^{t_m}\boldsymbol{\phi}_{ib}^b(t,t_{m-1})\times\boldsymbol{f}_{sf}^b(t)\,\mathrm{d}t \approx \int_{t_{m-1}}^{t_m}\boldsymbol{\theta}_{ib}^b(t,t_{m-1})\times\boldsymbol{f}_{sf}^b(t)\,\mathrm{d}t$ 等效旋转矢量等效成角增量

若计算 $\frac{\mathrm{d}[\boldsymbol{\theta}_{ib}^b(t,t_{m-1})\times\boldsymbol{v}_{sf}^b(t,t_{m-1})]}{\mathrm{d}t} = \boldsymbol{\omega}_{ib}^b(t)\times\boldsymbol{v}_{sf}^b(t,t_{m-1}) + \boldsymbol{\theta}_{ib}^b(t)\times\boldsymbol{f}_{sf}^b(t)$

$$= \boldsymbol{\omega}_{ib}^b(t)\times\boldsymbol{v}_{sf}^b(t) - \boldsymbol{\theta}_{ib}^b(t,t_{m-1})\times\boldsymbol{f}_{sf}^b(t) + 2\boldsymbol{\theta}_{ib}^b(t,t_{m-1})\times\boldsymbol{f}_{sf}^b(t)$$

上式移项则有

$$\boldsymbol{\theta}_{ib}^b(t,t_{m-1})\times\boldsymbol{f}_{sf}^b(t) = \frac{1}{2}\frac{\mathrm{d}[\boldsymbol{\theta}_{ib}^b(t,t_{m-1})\times\boldsymbol{v}_{sf}^b(t,t_{m-1})]}{\mathrm{d}t} + \frac{1}{2}\left[\boldsymbol{\theta}_{ib}^b(t,t_{m-1})\times\boldsymbol{f}_{sf}^b(t) + \boldsymbol{v}_{sf}^b(t,t_{m-1})\times\boldsymbol{\omega}_{ib}^b(t)\right]$$

上式积分有

$$\boldsymbol{\theta}_{ib}^b(t,t_{m-1})\times\boldsymbol{f}_{sf}^b(t) = \frac{\mathrm{d}[\boldsymbol{\theta}_{ib}^b(t,t_{m-1})\times\boldsymbol{v}_{sf}^b(t,t_{m-1})]}{\mathrm{d}t} + \boldsymbol{v}_{sf}^b(t,t_{m-1})\times\boldsymbol{\omega}_{ib}^b(t) \quad ?$$

$$\int_{t_{m-1}}^{t_m}\boldsymbol{\theta}_{ib}^b(t,t_{m-1})\times\boldsymbol{f}_{sf}^b(t)\,\mathrm{d}t$$

简单→复杂

$$= \frac{1}{2}\boldsymbol{\theta}_{ib}^b(t_m,t_{m-1})\times\boldsymbol{v}_{sf}^b(t_m,t_{m-1}) + \frac{1}{2}\int_{t_{m-1}}^{t_m}\boldsymbol{\theta}_{ib}^b(t,t_{m-1})\times\boldsymbol{f}_{sf}^b(t) + \boldsymbol{v}_{sf}^b(t,t_{m-1})\times\boldsymbol{\omega}_{ib}^b(t)\,\mathrm{d}t$$

$$= \Delta\boldsymbol{v}_{rot(m)}^{b(m-1)} + \Delta\boldsymbol{v}_{scul(m)}^{b(m-1)}$$ 旋转效应补偿+划桨效应补偿

已经积完，传感器可测出

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$$\Delta\boldsymbol{v}_{scul(m)}^{b(m-1)} = \frac{1}{2}\int_{t_{m-1}}^{t_m}\boldsymbol{\theta}_{ib}^b(t,t_{m-1})\times\boldsymbol{f}_{sf}^b(t) + \boldsymbol{v}_{sf}^b(t,t_{m-1})\times\boldsymbol{\omega}_{ib}^b(t)\,\mathrm{d}t$$

## 3 捷联惯导更新算法

基于多项式拟合的划桨效应补偿算法

假设运动速率的多项式形式

$$\begin{cases}\boldsymbol{\omega}_{ib}^b(t) = \boldsymbol{a} + 2\boldsymbol{b}(t-t_{m-1}) \\ \boldsymbol{f}_{sf}^b(t) = \boldsymbol{A} + 2\boldsymbol{B}(t-t_{m-1})\end{cases}$$

则运动增量的多项式形式为

$$\begin{cases}\boldsymbol{\theta}_{ib}^b(t,t_{m-1}) = \int_{t_{m-1}}^{t}\boldsymbol{\omega}_{ib}^b(\tau)\,\mathrm{d}\tau = \boldsymbol{a}(t-t_{m-1}) + \boldsymbol{b}(t-t_{m-1})^2 \\ \boldsymbol{v}_{sf}^b(t,t_{m-1}) = \int_{t_{m-1}}^{t}\boldsymbol{f}_{sf}^b(\tau)\,\mathrm{d}\tau = \boldsymbol{A}(t-t_{m-1}) + \boldsymbol{B}(t-t_{m-1})^2\end{cases}$$

陀螺/加计增量采样

$$\begin{cases}\Delta\boldsymbol{\theta}_{m1} = \int_{t_{m-1}}^{t_{m-1}+h}\boldsymbol{\omega}_{ib}^b(\tau)\,\mathrm{d}\tau = h\boldsymbol{a} + h^2\boldsymbol{b} \\ \Delta\boldsymbol{\theta}_{m2} = \int_{t_{m-1}+h}^{t_m}\boldsymbol{\omega}_{ib}^b(\tau)\,\mathrm{d}\tau = h\boldsymbol{a} + 3h^2\boldsymbol{b} \\ \Delta\boldsymbol{v}_{m1} = \int_{t_{m-1}}^{t_{m-1}+h}\boldsymbol{f}_{sf}^b(\tau)\,\mathrm{d}\tau = h\boldsymbol{A} + h^2\boldsymbol{B} \\ \Delta\boldsymbol{v}_{m2} = \int_{t_{m-1}+h}^{t_m}\boldsymbol{f}_{sf}^b(\tau)\,\mathrm{d}\tau = h\boldsymbol{A} + 3h^2\boldsymbol{B} \\ \boldsymbol{\theta}_m = \boldsymbol{\theta}_{ib}^b(t_m,t_{m-1}) = \int_{t_{m-1}}^{t_{m-1}+2h}\boldsymbol{\omega}_{ib}^b(\tau)\,\mathrm{d}\tau = \Delta\boldsymbol{\theta}_{m1} + \Delta\boldsymbol{\theta}_{m2} \\ \boldsymbol{v}_m = \boldsymbol{v}_{sf}^b(t_m,t_{m-1}) = \int_{t_{m-1}}^{t_{m-1}+2h}\boldsymbol{f}_{sf}^b(\tau)\,\mathrm{d}\tau = \Delta\boldsymbol{v}_{m1} + \Delta\boldsymbol{v}_{m2}\end{cases}$$

可得划桨误差二子样补偿算法

$$\Delta\boldsymbol{v}_{scul(m)}^{b(m-1)} = \frac{1}{2}\int_{t_{m-1}}^{t_m}\boldsymbol{\theta}_{ib}^b(t,t_{m-1})\times\boldsymbol{f}_{sf}^b(t) + \boldsymbol{v}_{sf}^b(t,t_{m-1})\times\boldsymbol{\omega}_{ib}^b(t)\,\mathrm{d}t$$

$$= \frac{1}{2}\int_{t_{m-1}}^{t_m}(\boldsymbol{a}\times\boldsymbol{B} + \boldsymbol{A}\times\boldsymbol{b})(t-t_{m-1})^2\,\mathrm{d}t = (\boldsymbol{a}\times\boldsymbol{B} + \boldsymbol{A}\times\boldsymbol{b})\frac{(2h)^3}{6}$$

$$= \frac{2}{3}(\Delta\boldsymbol{\theta}_{m1}\times\Delta\boldsymbol{v}_{m2} + \Delta\boldsymbol{v}_{m1}\times\Delta\boldsymbol{\theta}_{m2})$$

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## 3 捷联惯导更新算法

（3）划桨补偿算法与圆锥算法之间的对偶性

http://fz.pyvtc.edu.cn/info/1326/4492.htm

角运动为绕x振动 $\boldsymbol{\omega}_{ib}^{b}(t)=\begin{bmatrix} a\Omega\sin\Omega t \\ 0 \\ 0 \end{bmatrix}$ $\boldsymbol{\theta}_{ib}^{b}(t,t_{m-1})=\begin{bmatrix} -a(\cos\Omega t-\cos\Omega t_{m-1}) \\ 0 \\ 0 \end{bmatrix}$

线运动为绕y振动 $\boldsymbol{f}_{sf}^{b}(t)=\begin{bmatrix} 0 \\ \beta\Omega\cos\Omega t \\ 0 \end{bmatrix}$ $\boldsymbol{v}_{sf}^{b}(t,t_{m-1})=\begin{bmatrix} 0 \\ \beta(\sin\Omega t-\sin\Omega t_{m-1}) \\ 0 \end{bmatrix}$

$$\Delta\boldsymbol{v}_{scul(m)}^{b(m-1)}=\frac{1}{2}\int_{t_{m-1}}^{t_m}\boldsymbol{\theta}_{ib}^{b}(t,t_{m-1})\times\boldsymbol{f}_{sf}^{b}(t)+\boldsymbol{v}_{sf}^{b}(t,t_{m-1})\times\boldsymbol{\omega}_{ib}^{b}(t)\mathrm{d}t$$

$$=\frac{1}{2}\int_{t_{m-1}}^{t_m}\begin{bmatrix} -a(\cos\Omega t-\cos\Omega t_{m-1}) \\ 0 \\ 0 \end{bmatrix}\times\begin{bmatrix} 0 \\ \beta\Omega\cos\Omega t \\ 0 \end{bmatrix}+\begin{bmatrix} 0 \\ \beta(\sin\Omega t-\sin\Omega t_{m-1}) \\ 0 \end{bmatrix}\times\begin{bmatrix} a\Omega\sin\Omega t \\ 0 \\ 0 \end{bmatrix}\mathrm{d}t$$

$$=\frac{1}{2}\int_{t_{m-1}}^{t_m}\begin{bmatrix} -a(\cos\Omega t-\cos\Omega t_{m-1}) \\ \beta(\sin\Omega t-\sin\Omega t_{m-1}) \\ 0 \end{bmatrix}\times\begin{bmatrix} a\Omega\sin\Omega t \\ \beta\Omega\cos\Omega t \\ 0 \end{bmatrix}\mathrm{d}t$$

$$=\frac{1}{2}\int_{t_{m-1}}^{t_m}\left[\boldsymbol{\theta}_{ib}^{b}(t,t_{m-1})+\boldsymbol{v}_{sf}^{b}(t,t_{m-1})\right]\times\left[\boldsymbol{\omega}_{ib}^{b}(t)+\boldsymbol{f}_{sf}^{b}(t)\right]\mathrm{d}t$$

V.S.

$$\boldsymbol{\sigma}(t_m,t_{m-1})=\frac{1}{2}\int_{t_{m-1}}^{t_m}\Delta\boldsymbol{\theta}(\tau,t_{m-1})\times\boldsymbol{\omega}(\tau)\mathrm{d}\tau$$

$$=\frac{1}{2}\int_{t_{m-1}}^{t_m}\begin{bmatrix} -a(\cos\Omega\tau-\cos\Omega t_{m-1}) \\ b(\sin\Omega\tau-\sin\Omega t_{m-1}) \\ 0 \end{bmatrix}\times\begin{bmatrix} a\Omega\sin\Omega\tau \\ b\Omega\cos\Omega\tau \\ 0 \end{bmatrix}\mathrm{d}\tau$$

数学上形式相同

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## 3 捷联惯导更新算法

（4）速度更新算法小结

$$\dot{\boldsymbol{v}}^{n}(t)=\boldsymbol{C}_{b}^{n}(t)\boldsymbol{f}_{sf}^{b}(t)-\{2[\boldsymbol{\omega}_{ie}^{n}(t)\times]+[\boldsymbol{\omega}_{en}^{n}(t)\times]\}\boldsymbol{v}^{n}(t)+\boldsymbol{g}^{n}(t)$$

$$\Rightarrow \boldsymbol{v}_{m}^{n(m)}=\boldsymbol{v}_{m-1}^{n(m-1)}+\Delta\boldsymbol{v}_{sf(m)}^{n}+\Delta\boldsymbol{v}_{cor/g(m)}^{n}$$

$$\Delta\boldsymbol{v}_{sf(m)}^{n}=\boldsymbol{C}_{b(m-1)}^{n(m-1)}\Delta\boldsymbol{v}_{sf(m)}^{b}$$ 比力增量 低精度此项一项即可

$$-\frac{T_m}{2}[\boldsymbol{\omega}_{in}^{n}(t_{m-1/2})\times]\boldsymbol{C}_{b(m-1)}^{n(m-1)}\Delta\boldsymbol{v}_{sf}^{b}(t_m)$$ 导航系旋转修正

$$+\boldsymbol{C}_{b(m-1)}^{n(m-1)}\left[\frac{1}{2}\Delta\boldsymbol{\theta}_m\times\Delta\boldsymbol{v}_m\right.$$ 旋转效应补偿

$$\left.+\frac{2}{3}(\Delta\boldsymbol{\theta}_{m1}\times\Delta\boldsymbol{v}_{m2}+\Delta\boldsymbol{v}_{m1}\times\Delta\boldsymbol{\theta}_{m2})\right]$$ 划桨效应补偿

快变部分

$$\Delta\boldsymbol{v}_{cor/g(m)}^{n}=-\left\{\{2[\boldsymbol{\omega}_{ie}^{n}(t_{m-1/2})\times]+[\boldsymbol{\omega}_{en}^{n}(t_{m-1/2})\times]\}\boldsymbol{v}^{n}(t_{m-1/2})+\boldsymbol{g}^{n}(t_{m-1/2})\right\}T_m$$ 慢变部分

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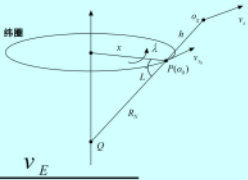
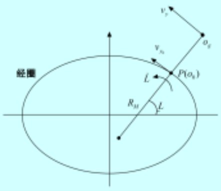
# 3 捷联惯导更新算法





## 3.3 位置更新

地标得

位置微分方程
$$\begin{cases} \dot{\lambda} = \dfrac{v_E}{(R_N + h)\cos L} \\ \dot{L} = \dfrac{v_N}{R_M + h}, \qquad \dot{h} = v_U \end{cases}$$

简记为 $\dot{\boldsymbol{p}} = \boldsymbol{M}_{pv}\boldsymbol{v}^n$ $\qquad \boldsymbol{p} = \begin{bmatrix} L \\ \lambda \\ h \end{bmatrix} \qquad \boldsymbol{M}_{pv} = \begin{bmatrix} 0 & 1/R_{Mh} & 0 \\ \sec L / R_{Nh} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

上式两边积分，得

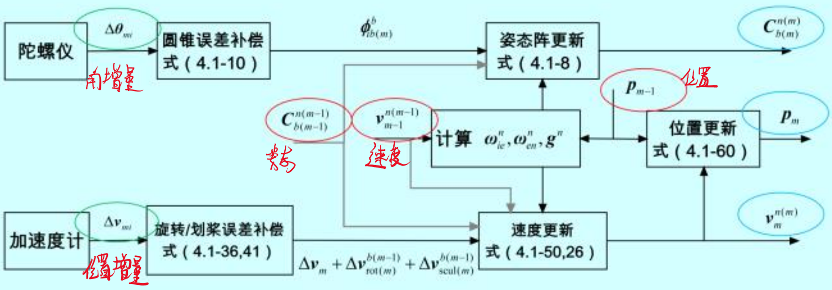
$$\boldsymbol{p}_m - \boldsymbol{p}_{m-1} = \int_{t_{m-1}}^{t_m} \boldsymbol{M}_{pv}\boldsymbol{v}^n \mathrm{d}t \approx \boldsymbol{M}_{pv}(t_{m-1/2})\int_{t_{m-1}}^{t_m} \boldsymbol{v}^n \mathrm{d}t \approx \boldsymbol{M}_{pv(m-1/2)}(\boldsymbol{v}_{m-1}^n + \boldsymbol{v}_m^n)\frac{T_m}{2}$$ 梯形积分

整理得位置更新 $\boldsymbol{p}_m = \boldsymbol{p}_{m-1} + \boldsymbol{M}_{pv(m-1/2)}(\boldsymbol{v}_{m-1}^n + \boldsymbol{v}_m^n)\dfrac{T_m}{2}$



# 3 捷联惯导更新算法

## 3.4 捷联惯导数值更新全套算法总结




三种导航信息初值 $\xrightarrow[\text{增量数据输入}]{\text{两类传感器}}$ 三种导航信息输出

$$\boldsymbol{v}_m^{n(m)} = \boldsymbol{v}_{m-1}^{n(m-1)} + \Delta\boldsymbol{v}_{sf(m)}^{n} + \Delta\boldsymbol{v}_{cor/g(m)}^{n}$$

$$\Delta\boldsymbol{v}_{sf(m)}^{n} = \left[\boldsymbol{I} - \frac{T}{2}(\boldsymbol{\omega}_{in(m-1/2)}^{n}\times)\right]\boldsymbol{C}_{b(m-1)}^{n(m-1)}(\Delta\boldsymbol{v}_m + \Delta\boldsymbol{v}_{rot(m)}^{b(m-1)} + \Delta\boldsymbol{v}_{scul(m)}^{b(m-1)})$$

# 3 捷联惯导更新算法

$$\boldsymbol{p}_m = \boldsymbol{p}_{m-1} + \boldsymbol{M}_{pv(m-1/2)}(\boldsymbol{v}_{m-1}^{n} + \boldsymbol{v}_m^{n})\frac{T_m}{2}$$

## Matlab程序综合实现

$$\boldsymbol{C}_{b(m)}^{n(m)} = \boldsymbol{C}_{n(m-1)}^{n(m)}\boldsymbol{C}_{b(m-1)}^{n(m-1)}\boldsymbol{C}_{b(m)}^{b(m-1)}$$

```
function [qnb, vn, pos] = insupdate(qnb, vn, pos, wm, vm, ts)
   nn = size(wm,1);  nts = nn*ts;
   [phim, dvbm] = cnscl(wm, vm);  % 圆锥/划桨补偿
   eth = earth(pos, vn);  % 地球相关参数计算
   vn1 = vn + rv2m(-eth.wnin*nts/2)*qmulv(qnb,dvbm) + eth.gcc*nts;  % 速度更新
   vn = (vn+vn1)/2;
   pos = pos + [vn(2)/eth.RMh;vn(1)/eth.clRNh;vn(3)]*nts;  vn = vn1;  % 位置更新
   qnb = qmul(rv2q(-eth.wnin*nts), qmul(qnb, rv2q(phim)));  % 姿态更新
   qnb = qnormlz(qnb);
```

```
function [phim, dvbm] = cnscl(wm, vm)
  cs = [  [2,     0,     0,       0,      0       ]/3
          [9,     27,    0,       0,      0       ]/20
          [54,    92,    214,     0,      0       ]/105
          [250,   525,   650,     1375,   0       ]/504
          [2315, 4558, 7296, 7834, 15797]/4620  ]; % 补偿系数
  wmm = sum(wm,1);  vmm = sum(vm,1);
  dphim = zeros(1,3); scullm = zeros(1,3);
  n = size(wm, 1);  % 子样数
  if n>1
     csw=cs(n-1,1:n-1)*wm(1:n-1,:); csv=cs(n-1,1:n-1)*vm(1:n-1,:);
     dphim = cross(csw,wm(n,:));  % 圆锥补偿
     scullm = cross(csw,vm(n,:))+cross(csv,wm(n,:));  % 划桨补偿
  end
  phim = (wmm+dphim)';
  dvbm = (vmm+0.5*cross(wmm,vmm)+scullm)';
```

```
function eth = earth(pos, vn)  % 地球相关参数计算
global Re ff wie g0
   ee = sqrt(2*ff-ff^2);  e2 = ee^2;  % 第一偏心率
   eth.sl = sin(pos(1));  eth.cl = cos(pos(1));  eth.tl = eth.sl/eth.cl;
   eth.sl2 = eth.sl*eth.sl;  sl4 = eth.sl2*eth.sl2;
   sq = 1-e2*eth.sl2;  sq2 = sqrt(sq);
   eth.RMh = Re*(1-e2)/sq/sq2+pos(3);
   eth.RNh = Re/sq2+pos(3);  eth.clRNh = eth.cl*eth.RNh;
   eth.wnie = wie*[0; eth.cl; eth.sl];
   eth.vn = vn;
   eth.wnen = [-vn(2)/eth.RMh; vn(1)/eth.RNh; vn(1)/eth.RNh*eth.tl];
   eth.wnin = eth.wnie + eth.wnen;
   eth.wnien = eth.wnie + eth.wnin;
   gLh = g0*(1+5.27094e-3*eth.sl2+2.32718e-5*sl4)-3.086e-6*pos(3)
   eth.gn = [0;0;-gLh];
   eth.gcc = eth.gn - cross(eth.wnien,vn); % 考虑重力/哥氏力/向心力
```

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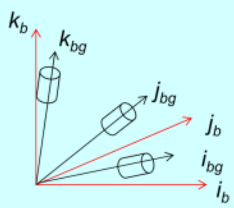
# 4 捷联惯导误差传播模型

## 4.1 惯性组件测量误差模型




单轴陀螺测量模型

$$\underbrace{\omega}_{\text{理论值}} = (1-\underbrace{\delta k}_{\text{刻度误差}})\underbrace{\tilde{\omega}}_{\text{测量值}} - \underbrace{\varepsilon}_{\text{零漂误差}}$$

陀螺三组件(triad)误差模型 (安装误差)

安装系

$$\begin{bmatrix} \omega_{ibx}^{b} \\ \omega_{iby}^{b} \\ \omega_{ibz}^{b} \end{bmatrix} = (\boldsymbol{C}_{b_g}^{b})^{-\mathrm{T}} \begin{bmatrix} (1-\delta k_{gxx})\omega_{ibx}^{b_g} - \varepsilon_x^{b_g} \\ (1-\delta k_{gyy})\omega_{iby}^{b_g} - \varepsilon_y^{b_g} \\ (1-\delta k_{gzz})\omega_{ibz}^{b_g} - \varepsilon_z^{b_g} \end{bmatrix}$$

理想系

$$\boldsymbol{\omega}_{ib}^{b} = (\boldsymbol{C}_{b_g}^{b})^{-\mathrm{T}}\left\{\left[\boldsymbol{I} - \mathrm{diag}(\delta\boldsymbol{k}_g)\right]\boldsymbol{\omega}_{ib}^{b_g} - \boldsymbol{\varepsilon}^{b_g}\right\}$$

令 $(\boldsymbol{I} - \delta\boldsymbol{K}_G)\boldsymbol{\omega}_{ib}^{b_g} - \boldsymbol{\varepsilon}^{b}$

$$\delta\boldsymbol{\omega}_{ib}^{b} = \boldsymbol{\omega}_{ib}^{b_g} - \boldsymbol{\omega}_{ib}^{b} \approx \delta\boldsymbol{K}_G\boldsymbol{\omega}_{ib}^{b} + \boldsymbol{\varepsilon}^{b}$$
$$= \omega_{ibx}^{b}\delta\boldsymbol{K}_{Gx} + \omega_{iby}^{b}\delta\boldsymbol{K}_{Gy} + \omega_{ibz}^{b}\delta\boldsymbol{K}_{Gz} + \boldsymbol{\varepsilon}^{b}$$

同理，可得加计三组件误差模型

$$\delta\boldsymbol{f}_{sf}^{b} = \boldsymbol{f}_{sf}^{b_g} - \boldsymbol{f}_{sf}^{b}$$
$$= f_{sfx}^{b}\delta\boldsymbol{K}_{Ax} + f_{sfy}^{b}\delta\boldsymbol{K}_{Ay} + f_{sfz}^{b}\delta\boldsymbol{K}_{Az} + \boldsymbol{\nabla}^{b}$$

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$$\dot{C}_b^n = C_b^n(\omega_{ib}^b\times) - (\omega_{in}^n\times)C_b^n$$

$$(V_1\times)(V_2\times) - (V_2\times)(V_1\times) = [(V_1\times V_2)\times]$$

## 4 捷联惯导误差传播模型

### 4.2 姿态(失准角)误差方程

真实姿态阵为 $C_b^n$ ,计算姿态阵为 $\tilde{C}_b^n$（常记为 $C_b^{n'}$）,有关系式 →认为b系相同，n系不重合有误差（  ）

等角度误差同理

$$C_b^{n'} = C_n^{n'} C_b^n \qquad n' \text{ 为计算导航坐标系，近似有 } C_{n'}^n \approx I + (\phi\times)$$

$$\left.\begin{aligned} C_b^{n'} &= [I-(\phi\times)]C_b^n \\ \dot{C}_b^{n'} &= C_b^{n'}(\tilde{\omega}_{ib}^b\times) - (\tilde{\omega}_{in}^n\times)C_b^{n'} \end{aligned}\right\} (-\dot{\phi}\times)C_b^n + (I-\phi\times)\dot{C}_b^n = C_b^{n'}(\tilde{\omega}_{ib}^b\times) - (\tilde{\omega}_{in}^n\times)C_b^{n'}$$

→微分

代入得 $(-\dot{\phi}\times)C_b^n + (I-\phi\times)\left[C_b^n(\omega_{ib}^b\times) - (\omega_{in}^n\times)C_b^n\right]$

$$= (I-\phi\times)C_b^n\left[(\omega_{ib}^b + \delta\omega_{ib}^b)\times\right] - \left[(\omega_{in}^n + \delta\omega_{in}^n)\times\right](I-\phi\times)C_b^n$$

整理得 $(\dot{\phi}\times) = \left[(\phi\times)(\omega_{in}^n\times) - (\omega_{in}^n\times)(\phi\times)\right] + (\delta\omega_{in}^n\times) - C_b^n(\delta\omega_{ib}^b\times)C_n^b$

$$= \left[(\phi\times\omega_{in}^n)\times\right] + (\delta\omega_{in}^n\times) - (\delta\omega_{ib}^n\times) = \left[(\phi\times\omega_{in}^n + \delta\omega_{in}^n - \delta\omega_{ib}^n)\times\right]$$

上式逆反对称化得 $\dot{\phi} = \phi\times\omega_{in}^n + \delta\omega_{in}^n - \delta\omega_{ib}^n$

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## 4 捷联惯导误差传播模型

### 4.3 速度误差方程

比力方程的理论公式 $\dot{v}^n = C_b^n f_{sf}^b - (2\omega_{ie}^n + \omega_{en}^n)\times v^n + g^n$

实际计算公式 $\dot{\tilde{v}}^n = \tilde{C}_b^n \tilde{f}_{sf}^b - (2\tilde{\omega}_{ie}^n + \tilde{\omega}_{en}^n)\times\tilde{v}^n + \tilde{g}^n$

两式相减，记误差

$$\delta\dot{v}^n = \dot{\tilde{v}}^n - \dot{v}^n$$

$$= (\tilde{C}_b^n\tilde{f}_{sf}^b - C_b^n f_{sf}^b) - \left[(2\tilde{\omega}_{ie}^n + \tilde{\omega}_{en}^n)\times\tilde{v}^n - (2\omega_{ie}^n + \omega_{en}^n)\times v^n\right] + (\tilde{g}^n - g^n)$$

$$= \left[(I-\phi\times)C_b^n(f_{sf}^b + \delta f_{sf}^b) - C_b^n f_{sf}^b\right]$$

$$-\left\{\left[2(\omega_{ie}^n + \delta\omega_{ie}^n) + (\omega_{en}^n + \delta\omega_{en}^n)\right]\times(v^n + \delta v^n) - (2\omega_{ie}^n + \omega_{en}^n)\times v^n\right\} + \delta g^n$$

$$\approx -(\phi\times)C_b^n f_{sf}^b + C_b^n\delta f_{sf}^b - (2\delta\omega_{ie}^n + \delta\omega_{en}^n)\times v^n - (2\omega_{ie}^n + \omega_{en}^n)\times\delta v^n + \delta g^n$$

由此得速度误差方程

$$\delta\dot{v}^n = f_{sf}^n\times\phi + v^n\times(2\delta\omega_{ie}^n + \delta\omega_{en}^n) - (2\omega_{ie}^n + \omega_{en}^n)\times\delta v^n + \delta f_{sf}^n + \delta g^n$$

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## 4 捷联惯导误差传播模型

### 4.4 位置误差方程

直接由位置更新微分方程

$$\dot{L}=\frac{1}{R_M+h}v_N,\quad \dot{\lambda}=\frac{\sec L}{R_N+h}v_E,\quad \dot{h}=v_U$$

求误差(短时内将 $R_M$ 和 $R_N$ 视为常值)，得

$$\begin{cases}\delta\dot{L}=\dfrac{1}{R_M+h}\delta v_N-\dfrac{v_N}{(R_M+h)^2}\delta h\\ \delta\dot{\lambda}=\dfrac{\sec L}{R_N+h}\delta v_E+\dfrac{v_E\sec L\tan L}{R_N+h}\delta L-\dfrac{v_E\sec L}{(R_N+h)^2}\delta h\\ \delta\dot{h}=\delta v_U\end{cases}$$

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## 4 捷联惯导误差传播模型

### 4.5 捷联惯导误差方程总结

$$\dot{\boldsymbol{\phi}}=\boldsymbol{M}_{aa}\boldsymbol{\phi}+\boldsymbol{M}_{av}\delta\boldsymbol{v}^n+\boldsymbol{M}_{ap}\delta\boldsymbol{p}-\omega_{ibx}^b\boldsymbol{C}_b^n\delta\boldsymbol{K}_{Gx}-\omega_{iby}^b\boldsymbol{C}_b^n\delta\boldsymbol{K}_{Gy}-\omega_{ibz}^b\boldsymbol{C}_b^n\delta\boldsymbol{K}_{Gz}-\boldsymbol{C}_b^n\boldsymbol{\varepsilon}^b$$

$$\delta\dot{\boldsymbol{v}}^n=\boldsymbol{M}_{va}\boldsymbol{\phi}+\boldsymbol{M}_{vv}\delta\boldsymbol{v}^n+\boldsymbol{M}_{vp}\delta\boldsymbol{p}+f_{sfx}^b\boldsymbol{C}_b^n\delta\boldsymbol{K}_{Ax}+f_{sfy}^b\boldsymbol{C}_b^n\delta\boldsymbol{K}_{Ay}+f_{sfz}^b\boldsymbol{C}_b^n\delta\boldsymbol{K}_{Az}+\boldsymbol{C}_b^n\boldsymbol{\nabla}^b$$

$$\delta\dot{\boldsymbol{p}}=\boldsymbol{M}_{pv}\delta\boldsymbol{v}+\boldsymbol{M}_{pp}\delta\boldsymbol{p}$$

其用途广泛：惯导误差传播规律分析、可观测性分析、系统级标定、初始对准、组合导航… 陀螺常值漂移，零偏

去掉后

(安装精确，陀螺刻度精准)

典型的15维误差传播线性状态方程 $\dot{\boldsymbol{X}}=\boldsymbol{F}\boldsymbol{X}$

其中 $\boldsymbol{X}=\begin{bmatrix}\boldsymbol{\phi}^{\mathrm{T}} & (\delta\boldsymbol{v}^n)^{\mathrm{T}} & (\delta\boldsymbol{p})^{\mathrm{T}} & (\boldsymbol{\varepsilon}^b)^{\mathrm{T}} & (\boldsymbol{\nabla}^b)^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$

$$\boldsymbol{F}=\begin{bmatrix}\boldsymbol{M}_{aa} & \boldsymbol{M}_{av} & \boldsymbol{M}_{ap} & -\boldsymbol{C}_b^n & \boldsymbol{0}_{3\times3}\\ \boldsymbol{M}_{va} & \boldsymbol{M}_{vv} & \boldsymbol{M}_{vp} & \boldsymbol{0}_{3\times3} & \boldsymbol{C}_b^n\\ \boldsymbol{0}_{3\times3} & \boldsymbol{M}_{pv} & \boldsymbol{M}_{pp} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3}\\ \multicolumn{5}{c}{\boldsymbol{0}_{6\times15}}\end{bmatrix}$$

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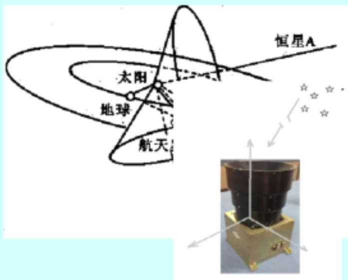
# 5 捷联惯导初始对准

## 5.1 矢量定姿原理（双矢量）




不同坐标系下观测同一矢量 $\boldsymbol{V}_1^r = \boldsymbol{C}_b^r \boldsymbol{V}_1^b$

第二矢量 $\boldsymbol{V}_2^r = \boldsymbol{C}_b^r \boldsymbol{V}_2^b$

构造第三辅助矢量

$$\boldsymbol{V}_1^r \times \boldsymbol{V}_2^r = (\boldsymbol{C}_b^r \boldsymbol{V}_1^b) \times (\boldsymbol{C}_b^r \boldsymbol{V}_2^b) = \boldsymbol{C}_b^r (\boldsymbol{V}_1^b \times \boldsymbol{V}_2^b)$$

联立 $\begin{bmatrix} \boldsymbol{V}_1^r & \boldsymbol{V}_2^r & \boldsymbol{V}_1^r \times \boldsymbol{V}_2^r \end{bmatrix} = \boldsymbol{C}_b^r \begin{bmatrix} \boldsymbol{V}_1^b & \boldsymbol{V}_2^b & \boldsymbol{V}_1^b \times \boldsymbol{V}_2^b \end{bmatrix}$

求得 $\boldsymbol{C}_b^r = \begin{bmatrix} \boldsymbol{V}_1^r & \boldsymbol{V}_2^r & \boldsymbol{V}_1^r \times \boldsymbol{V}_2^r \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_1^b & \boldsymbol{V}_2^b & \boldsymbol{V}_1^b \times \boldsymbol{V}_2^b \end{bmatrix}^{-1} = \begin{bmatrix} (\boldsymbol{V}_1^r)^{\mathrm{T}} \\ (\boldsymbol{V}_2^r)^{\mathrm{T}} \\ (\boldsymbol{V}_1^r \times \boldsymbol{V}_2^r)^{\mathrm{T}} \end{bmatrix}^{-1} \begin{bmatrix} (\boldsymbol{V}_1^b)^{\mathrm{T}} \\ (\boldsymbol{V}_2^b)^{\mathrm{T}} \\ (\boldsymbol{V}_1^b \times \boldsymbol{V}_2^b)^{\mathrm{T}} \end{bmatrix}$

更常用的处理方法(自然满足单位正交化要求)：

自然而然、单位正交了

$$\hat{\boldsymbol{C}}_b^r = \begin{bmatrix} (\frac{\tilde{\boldsymbol{V}}_1^r}{|\tilde{\boldsymbol{V}}_1^r|})^{\mathrm{T}} \\ (\frac{\tilde{\boldsymbol{V}}_1^r \times \tilde{\boldsymbol{V}}_2^r}{|\tilde{\boldsymbol{V}}_1^r \times \tilde{\boldsymbol{V}}_2^r|})^{\mathrm{T}} \\ (\frac{\tilde{\boldsymbol{V}}_1^r \times \tilde{\boldsymbol{V}}_2^r \times \tilde{\boldsymbol{V}}_1^r}{|\tilde{\boldsymbol{V}}_1^r \times \tilde{\boldsymbol{V}}_2^r \times \tilde{\boldsymbol{V}}_1^r|})^{\mathrm{T}} \end{bmatrix}^{-1} \begin{bmatrix} (\frac{\tilde{\boldsymbol{V}}_1^b}{|\tilde{\boldsymbol{V}}_1^b|})^{\mathrm{T}} \\ (\frac{\tilde{\boldsymbol{V}}_1^b \times \tilde{\boldsymbol{V}}_2^b}{|\tilde{\boldsymbol{V}}_1^b \times \tilde{\boldsymbol{V}}_2^b|})^{\mathrm{T}} \\ (\frac{\tilde{\boldsymbol{V}}_1^b \times \tilde{\boldsymbol{V}}_2^b \times \tilde{\boldsymbol{V}}_1^b}{|\tilde{\boldsymbol{V}}_1^b \times \tilde{\boldsymbol{V}}_2^b \times \tilde{\boldsymbol{V}}_1^b|})^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} \frac{\tilde{\boldsymbol{V}}_1^r}{|\tilde{\boldsymbol{V}}_1^r|} & \frac{\tilde{\boldsymbol{V}}_1^r \times \tilde{\boldsymbol{V}}_2^r}{|\tilde{\boldsymbol{V}}_1^r \times \tilde{\boldsymbol{V}}_2^r|} & \frac{\tilde{\boldsymbol{V}}_1^r \times \tilde{\boldsymbol{V}}_2^r \times \tilde{\boldsymbol{V}}_1^r}{|\tilde{\boldsymbol{V}}_1^r \times \tilde{\boldsymbol{V}}_2^r \times \tilde{\boldsymbol{V}}_1^r|} \end{bmatrix} \begin{bmatrix} (\frac{\tilde{\boldsymbol{V}}_1^b}{|\tilde{\boldsymbol{V}}_1^b|})^{\mathrm{T}} \\ (\frac{\tilde{\boldsymbol{V}}_1^b \times \tilde{\boldsymbol{V}}_2^b}{|\tilde{\boldsymbol{V}}_1^b \times \tilde{\boldsymbol{V}}_2^b|})^{\mathrm{T}} \\ (\frac{\tilde{\boldsymbol{V}}_1^b \times \tilde{\boldsymbol{V}}_2^b \times \tilde{\boldsymbol{V}}_1^b}{|\tilde{\boldsymbol{V}}_1^b \times \tilde{\boldsymbol{V}}_2^b \times \tilde{\boldsymbol{V}}_1^b|})^{\mathrm{T}} \end{bmatrix}$$



# 5 捷联惯导初始对准

$$\left|\tilde{\boldsymbol{V}}_i^r - \boldsymbol{C}_b^r \tilde{\boldsymbol{V}}_i^b\right|^2 = (\tilde{\boldsymbol{V}}_i^r - \boldsymbol{C}_b^r \tilde{\boldsymbol{V}}_i^b)^{\mathrm{T}} (\tilde{\boldsymbol{V}}_i^r - \boldsymbol{C}_b^r \tilde{\boldsymbol{V}}_i^b)$$
$$= \left[(\tilde{\boldsymbol{V}}_i^r)^{\mathrm{T}} - (\tilde{\boldsymbol{V}}_i^b)^{\mathrm{T}} (\boldsymbol{C}_b^r)^{\mathrm{T}}\right] (\tilde{\boldsymbol{V}}_i^r - \boldsymbol{C}_b^r \tilde{\boldsymbol{V}}_i^b)$$
$$= \left|\tilde{\boldsymbol{V}}_i^r\right|^2 - (\tilde{\boldsymbol{V}}_i^r)^{\mathrm{T}} \boldsymbol{C}_b^r \tilde{\boldsymbol{V}}_i^b - (\tilde{\boldsymbol{V}}_i^b)^{\mathrm{T}} (\boldsymbol{C}_b^r)^{\mathrm{T}} \tilde{\boldsymbol{V}}_i^r + (\tilde{\boldsymbol{V}}_i^b)^{\mathrm{T}} (\boldsymbol{C}_b^r)^{\mathrm{T}} \boldsymbol{C}_b^r \tilde{\boldsymbol{V}}_i^b$$
$$= \left|\tilde{\boldsymbol{V}}_i^r\right|^2 + \left|\tilde{\boldsymbol{V}}_i^b\right|^2 - 2(\tilde{\boldsymbol{V}}_i^r)^{\mathrm{T}} \boldsymbol{C}_b^r \tilde{\boldsymbol{V}}_i^b$$

多矢量相当于方程比未知数多

矢量定姿原理（多矢量） 状态多维 (Z=HX)

第$i$个矢量观测 $\tilde{\boldsymbol{V}}_i^r \approx \boldsymbol{C}_b^r \tilde{\boldsymbol{V}}_i^b \quad (i = 1, 2, \cdots m)$

为单位正交

构造指标函数 $J^*(\boldsymbol{C}_b^r) = \frac{1}{2}\sum_{i=1}^m w_i \left|\tilde{\boldsymbol{V}}_i^r - \boldsymbol{C}_b^r \tilde{\boldsymbol{V}}_i^b\right|^2 = \min$ 　带约束的最小二乘问题 (Wahba问题)

$$= \frac{1}{2}\sum_{i=1}^m w_i (\left|\tilde{\boldsymbol{V}}_i^r\right|^2 + \left|\tilde{\boldsymbol{V}}_i^b\right|^2) - \sum_{i=1}^m w_i (\tilde{\boldsymbol{V}}_i^r)^{\mathrm{T}} \boldsymbol{C}_b^r \tilde{\boldsymbol{V}}_i^b$$

→观测量为常值

$$\Leftrightarrow J(\boldsymbol{C}_b^r) = \sum_{i=1}^m w_i (\tilde{\boldsymbol{V}}_i^r)^{\mathrm{T}} \boldsymbol{C}_b^r \tilde{\boldsymbol{V}}_i^b = \max$$

$$= \mathrm{tr}\left( \begin{bmatrix} w_1(\tilde{\boldsymbol{V}}_1^r)^{\mathrm{T}} \\ w_2(\tilde{\boldsymbol{V}}_2^r)^{\mathrm{T}} \\ \vdots \\ w_m(\tilde{\boldsymbol{V}}_m^r)^{\mathrm{T}} \end{bmatrix} \boldsymbol{C}_b^r \begin{bmatrix} \tilde{\boldsymbol{V}}_1^b & \tilde{\boldsymbol{V}}_2^b & \cdots & \tilde{\boldsymbol{V}}_m^b \end{bmatrix} \right) = \mathrm{tr}\left( \boldsymbol{C}_b^r \begin{bmatrix} \tilde{\boldsymbol{V}}_1^b & \tilde{\boldsymbol{V}}_2^b & \cdots & \tilde{\boldsymbol{V}}_m^b \end{bmatrix} \begin{bmatrix} w_1(\tilde{\boldsymbol{V}}_1^r)^{\mathrm{T}} \\ w_2(\tilde{\boldsymbol{V}}_2^r)^{\mathrm{T}} \\ \vdots \\ w_m(\tilde{\boldsymbol{V}}_m^r)^{\mathrm{T}} \end{bmatrix} \right)$$

$$= \mathrm{tr}(\boldsymbol{C}_b^r \sum_{i=1}^m w_i \tilde{\boldsymbol{V}}_i^b (\tilde{\boldsymbol{V}}_i^r)^{\mathrm{T}})$$

→观测量,所以为常值矩阵

记 $\boldsymbol{A} = \left[\sum_{i=1}^m w_i \tilde{\boldsymbol{V}}_i^b (\tilde{\boldsymbol{V}}_i^r)^{\mathrm{T}}\right]^{\mathrm{T}} = \sum_{i=1}^m w_i \tilde{\boldsymbol{V}}_i^r (\tilde{\boldsymbol{V}}_i^b)^{\mathrm{T}}$ 　其奇异值分解 $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{D}\boldsymbol{V}^{\mathrm{T}}$

$\Leftrightarrow$ 　$\mathrm{tr}(\boldsymbol{C}_b^r \boldsymbol{A}^{\mathrm{T}}) = \max$ 　　求得最优姿态阵 $\hat{\boldsymbol{C}}_b^r = \boldsymbol{U}\boldsymbol{V}^{\mathrm{T}}$

变优化问题了

## 5 捷联惯导初始对准

(1)SVD法 **传统稳健,计算量较大(1000)**

$J(C)=\mathrm{tr}(CA^{\mathrm{T}})=\max$

姿态阵最优单位正交化的其它方法：

$$\Leftrightarrow J(Q)=\mathrm{tr}\left(\left[(q_0^2-q_v^{\mathrm{T}}q_v)I+2q_vq_v^{\mathrm{T}}+2q_0(q_v\times)\right]A^{\mathrm{T}}\right)$$

$$z=\begin{bmatrix}A_{32}-A_{23}\\A_{13}-A_{31}\\A_{21}-A_{12}\end{bmatrix}$$

(2)QUEST法 **高大上,花瓶!**

QUaternion ESTmator

$$=\begin{bmatrix}q_0 & q_v^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}\mathrm{tr}(A) & z^{\mathrm{T}}\\ z & A+A^{\mathrm{T}}-\mathrm{tr}(A)I\end{bmatrix}\begin{bmatrix}q_0\\q_v\end{bmatrix}=Q^{\mathrm{T}}MQ=\max$$

四维

二次型 $Q^{\mathrm{T}}MQ=\max \quad \Rightarrow Q=V(\lambda_{\max})$ 矩阵$M$最大特征值对应的单位特征向量!

(3)FOAM法 采用牛顿迭代法 $\lambda_{k+1}=\lambda_k-\psi(\lambda_k)/\psi'(\lambda_k)$ 求解下式的最大根

Fast Optimal Attitude Matrix

为了解决计算量问题

特征多程

$$\psi(x)=\left(x^2-\|A\|^2\right)^2-8x|A|-4\left\|\left(A^{\mathrm{T}}\right)^*\right\|^2=0$$ **计算量最小(200)**

记 $\kappa=\left(\lambda^2-\|A\|^2\right)/2,\ \zeta=\kappa\lambda-|A|$ 则有 $C=\left[\left(\kappa+\|A\|^2\right)A+\lambda\left(A^{\mathrm{T}}\right)^*-AA^{\mathrm{T}}A\right]/\zeta$

(4)直接迭代法 **简单,条件数大时收敛慢,计算量较大(30*k)**

$$C_{k+1}=\frac{1}{2}(C_k+C_k^{-\mathrm{T}}) \quad 或 \quad C_{k+1}=\frac{2}{\mathrm{tr}(C_kC_k^{\mathrm{T}})+1}\left[C_k+(C_k^{\mathrm{T}})^*\right]$$

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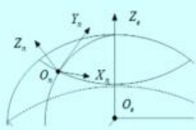
重力　地球自转角速率

## 5 捷联惯导初始对准

两个天然参考量 $g^n=\begin{bmatrix}0\\0\\-g\end{bmatrix}$ $\omega_{ie}^n=\begin{bmatrix}0\\\omega_{ie}\cos L\\\omega_{ie}\sin L\end{bmatrix}$

5.2 静基座初始粗对准（解析法）




静基座下的比力和角速度测量关系式

为0 ← $\dot{v}^n=C_b^nf_{sf}^b-(2\omega_{ie}^n+\omega_{en}^n)\times v^n+g^n$

$C_b^n\omega_{ib}^b=\omega_{ib}^n=\omega_{ie}^n+\omega_{en}^n+\omega_{nb}^n$

$$\Rightarrow\begin{cases}C_b^n\tilde{f}_{sf}^b=-g^n & V_1\\ C_b^n\tilde{\omega}_{ib}^b=\omega_{ie}^n & V_2\end{cases}$$

根据双矢量定姿原理，可得

$$\hat{C}_b^n=\begin{bmatrix}\frac{(-g^n)}{|(-g^n)|} & \frac{(-g^n)\times\omega_{ie}^n}{|(-g^n)\times\omega_{ie}^n|} & \frac{(-g^n)\times\omega_{ie}^n\times(-g^n)}{|(-g^n)\times\omega_{ie}^n\times(-g^n)|}\end{bmatrix}\times\begin{bmatrix}(\frac{\tilde{f}_{sf}^b}{|\tilde{f}_{sf}^b|})^{\mathrm{T}}\\(\frac{\tilde{f}_{sf}^b\times\tilde{\omega}_{ib}^b}{|\tilde{f}_{sf}^b\times\tilde{\omega}_{ib}^b|})^{\mathrm{T}}\\(\frac{\tilde{f}_{sf}^b\times\tilde{\omega}_{ib}^b\times\tilde{f}_{sf}^b}{|\tilde{f}_{sf}^b\times\tilde{\omega}_{ib}^b\times\tilde{f}_{sf}^b|})^{\mathrm{T}}\end{bmatrix}$$

$g=[0,0,g]^{\mathrm{T}}$

$$=\begin{bmatrix}0&-1&0\\0&0&1\\1&0&0\end{bmatrix}\begin{bmatrix}(\frac{\tilde{f}_{sf}^b}{|\tilde{f}_{sf}^b|})^{\mathrm{T}}\\(\frac{\tilde{f}_{sf}^b\times\tilde{\omega}_{ib}^b}{|\tilde{f}_{sf}^b\times\tilde{\omega}_{ib}^b|})^{\mathrm{T}}\\(\frac{\tilde{f}_{sf}^b\times\tilde{\omega}_{ib}^b\times\tilde{f}_{sf}^b}{|\tilde{f}_{sf}^b\times\tilde{\omega}_{ib}^b\times\tilde{f}_{sf}^b|})^{\mathrm{T}}\end{bmatrix}=\begin{bmatrix}-(\frac{\tilde{f}_{sf}^b\times\tilde{\omega}_{ib}^b}{|\tilde{f}_{sf}^b\times\tilde{\omega}_{ib}^b|})^{\mathrm{T}}\\(\frac{\tilde{f}_{sf}^b\times\tilde{\omega}_{ib}^b\times\tilde{f}_{sf}^b}{|\tilde{f}_{sf}^b\times\tilde{\omega}_{ib}^b\times\tilde{f}_{sf}^b|})^{\mathrm{T}}\\(\frac{\tilde{f}_{sf}^b}{|\tilde{f}_{sf}^b|})^{\mathrm{T}}\end{bmatrix}$$

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## 5 捷联惯导初始对准

解析法粗对准的误差分析

$$\hat{C}_b^n = \begin{bmatrix} -\left(\dfrac{\tilde{f}_{sf}^b \times \tilde{\omega}_{ib}^b}{\left|\tilde{f}_{sf}^b \times \tilde{\omega}_{ib}^b\right|}\right)^{\mathrm T} \\ \left(\dfrac{\tilde{f}_{sf}^b \times \tilde{\omega}_{ib}^b \times \tilde{f}_{sf}^b}{\left|\tilde{f}_{sf}^b \times \tilde{\omega}_{ib}^b \times \tilde{f}_{sf}^b\right|}\right)^{\mathrm T} \\ \left(\dfrac{\tilde{f}_{sf}^b}{\left|\tilde{f}_{sf}^b\right|}\right)^{\mathrm T} \end{bmatrix}$$

$$\hat{C}_b^n \approx \begin{bmatrix} -\left[(f_{sf}^b+\delta f_{sf}^b)\times(\omega_{ib}^b+\delta\omega_{ib}^b)\right]^{\mathrm T}/(g\omega_{\mathrm N}) \\ \left[(f_{sf}^b+\delta f_{sf}^b)\times(\omega_{ib}^b+\delta\omega_{ib}^b)\times(f_{sf}^b+\delta f_{sf}^b)\right]^{\mathrm T}/(g^2\omega_{\mathrm N}) \\ (f_{sf}^b+\delta f_{sf}^b)^{\mathrm T}/g \end{bmatrix}$$ ?

$$\approx \begin{bmatrix} -\left(f_{sf}^b\times\omega_{ib}^b\right)^{\mathrm T}/(g\omega_{\mathrm N}) \\ \left(f_{sf}^b\times\omega_{ib}^b\times f_{sf}^b\right)^{\mathrm T}/(g^2\omega_{\mathrm N}) \\ \left(f_{sf}^b\right)^{\mathrm T}/g \end{bmatrix} + \begin{bmatrix} -\left(\delta f_{sf}^b\times\omega_{ib}^b + f_{sf}^b\times\delta\omega_{ib}^b\right)^{\mathrm T}/(g\omega_{\mathrm N}) \\ * \\ \left(\delta f_{sf}^b\right)^{\mathrm T}/g \end{bmatrix} C_n^b C_b^n$$

$$= \left(I + \begin{bmatrix} * & -\delta\omega_{ib,\mathrm E}^n/\omega_{\mathrm N} + \delta f_{sf,\mathrm E}^n \tan L/g & * \\ * & * & * \\ \delta f_{sf,\mathrm E}^n/g & \delta f_{sf,\mathrm N}^n/g & * \end{bmatrix}\right) C_b^n$$

与 $\hat{C}_b^n = (I-\phi\times)C_b^n$ 对比可得 $\phi = \begin{bmatrix} -\delta f_{sf,\mathrm N}^n/g \\ \delta f_{sf,\mathrm E}^n/g \\ -\delta\omega_{ib,\mathrm E}^n/\omega_{\mathrm N} + \delta f_{sf,\mathrm E}^n \tan L/g \end{bmatrix} \approx \begin{bmatrix} -\delta f_{sf,\mathrm N}^n/g \\ \delta f_{sf,\mathrm E}^n/g \\ -\delta\omega_{ib,\mathrm E}^n/\omega_{\mathrm N} \end{bmatrix}$

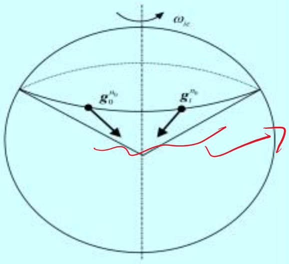
东
北
天



## 5 捷联惯导初始对准

5.3 晃动基座下的间接粗对准法

新定义两个惯性坐标系：初始时刻导航惯性系 $n_0$；初始时刻载体惯性系 $b_0$




惯性空间中观察
重力矢量形成锥面

两个时刻的動

由 $\dot{C}_n^{n_0} = C_n^{n_0}(\omega_{n_0 n}^n\times) = C_n^{n_0}(\omega_{ie}^n\times)$ 可解得

$$C_n^{n_0} = \mathrm{e}^{(t\omega_{ie}^n\times)} = I + \frac{\sin\omega_{ie}t}{\omega_{ie}t}(t\omega_{ie}^n\times) + \frac{1-\cos\omega_{ie}t}{(\omega_{ie}t)^2}(t\omega_{ie}^n\times)^2$$

$$= \begin{bmatrix} \cos\omega_{ie}t & -\sin\omega_{ie}t\sin L & \sin\omega_{ie}t\cos L \\ \sin\omega_{ie}t\sin L & 1-(1-\cos\omega_{ie}t)\sin^2 L & (1-\cos\omega_{ie}t)\sin L\cos L \\ -\sin\omega_{ie}t\cos L & (1-\cos\omega_{ie}t)\sin L\cos L & 1-(1-\cos\omega_{ie}t)\cos^2 L \end{bmatrix}$$

因而有

$$g^{n_0} = C_n^{n_0} g^n = -g\begin{bmatrix} \sin\omega_{ie}t\cos L \\ (1-\cos\omega_{ie}t)\sin L\cos L \\ 1-(1-\cos\omega_{ie}t)\cos^2 L \end{bmatrix}$$

## 5 捷联惯导初始对准

$$\boldsymbol{g}^{n_0}=-g\begin{bmatrix}\sin\omega_{ie}t\cos L\\(1-\cos\omega_{ie}t)\sin L\cos L\\1-(1-\cos\omega_{ie}t)\cos^2 L\end{bmatrix}$$

由 $\dot{\boldsymbol{C}}_b^{b_0}=\boldsymbol{C}_b^{b_0}(\boldsymbol{\omega}_{b_0b}^b\times)=\boldsymbol{C}_b^{b_0}(\boldsymbol{\omega}_{ib}^b\times)$ 可实时计算得 $\boldsymbol{C}_b^{b_0}$ 。

在角晃动基座下，该式依然成立 $\boldsymbol{C}_b^n\boldsymbol{f}_{sf}^b=-\boldsymbol{g}^n$

$\Rightarrow \boldsymbol{C}_n^{n_0}\boldsymbol{C}_b^n\boldsymbol{f}_{sf}^b=-\boldsymbol{C}_n^{n_0}\boldsymbol{g}^n \quad \Rightarrow \underline{\boldsymbol{C}_{b_0}^{n_0}\boldsymbol{C}_b^{b_0}\boldsymbol{f}_{sf}^b=-\boldsymbol{g}^{n_0}}$ 注意到 $\boldsymbol{C}_{b_0}^{n_0}$ 为常值矩阵

上式积分，在初始对准过程中的两个不同时刻取值并分别记为

$$\begin{cases}\tilde{\boldsymbol{F}}_1^{b_0}=\int_0^{t_1}\boldsymbol{C}_b^{b_0}\tilde{\boldsymbol{f}}_{sf}^b\mathrm{d}t & \boldsymbol{G}_1^{n_0}=-\int_0^{t_1}\boldsymbol{g}^{n_0}\mathrm{d}t\\ \tilde{\boldsymbol{F}}_2^{b_0}=\int_0^{t_2}\boldsymbol{C}_b^{b_0}\tilde{\boldsymbol{f}}_{sf}^b\mathrm{d}t & \boldsymbol{G}_2^{n_0}=-\int_0^{t_2}\boldsymbol{g}^{n_0}\mathrm{d}t\end{cases}$$

$\boldsymbol{Q}_{b_0}^{n_0}\circ(\boldsymbol{C}_b^{b_0}\boldsymbol{f}_{sf}^b+\mathrm{xxx})=(-\boldsymbol{g}^{n_0})\circ\boldsymbol{Q}_{b_0}^{n_0}$

也可以按多矢量定姿方法求解

根据双矢量定姿有 $\hat{\boldsymbol{C}}_{b_0}^{n_0}=\begin{bmatrix}\dfrac{\boldsymbol{G}_1^{n_0}}{\left|\boldsymbol{G}_1^{n_0}\right|} & \dfrac{\boldsymbol{G}_1^{n_0}\times\boldsymbol{G}_2^{n_0}}{\left|\boldsymbol{G}_1^{n_0}\times\boldsymbol{G}_2^{n_0}\right|} & \dfrac{\boldsymbol{G}_1^{n_0}\times\boldsymbol{G}_2^{n_0}\times\boldsymbol{G}_1^{n_0}}{\left|\boldsymbol{G}_1^{n_0}\times\boldsymbol{G}_2^{n_0}\times\boldsymbol{G}_1^{n_0}\right|}\end{bmatrix}\begin{bmatrix}\left(\dfrac{\tilde{\boldsymbol{F}}_1^{b_0}}{\left|\tilde{\boldsymbol{F}}_1^{b_0}\right|}\right)^{\mathrm{T}}\\ \left(\dfrac{\tilde{\boldsymbol{F}}_1^{b_0}\times\tilde{\boldsymbol{F}}_2^{b_0}}{\left|\tilde{\boldsymbol{F}}_1^{b_0}\times\tilde{\boldsymbol{F}}_2^{b_0}\right|}\right)^{\mathrm{T}}\\ \left(\dfrac{\tilde{\boldsymbol{F}}_1^{b_0}\times\tilde{\boldsymbol{F}}_2^{b_0}\times\tilde{\boldsymbol{F}}_1^{b_0}}{\left|\tilde{\boldsymbol{F}}_1^{b_0}\times\tilde{\boldsymbol{F}}_2^{b_0}\times\tilde{\boldsymbol{F}}_1^{b_0}\right|}\right)^{\mathrm{T}}\end{bmatrix}$

最后，可求得实时(对准结束时刻)的姿态阵

$$\hat{\boldsymbol{C}}_b^n=\boldsymbol{C}_{n_0}^n\hat{\boldsymbol{C}}_{b_0}^{n_0}\boldsymbol{C}_b^{b_0}$$

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## 5 捷联惯导初始对准

### 5.4 Kalman滤波精对准

静基座简化导航算法（仅需姿态和速度更新）位置不变

$$\begin{cases}\dot{\boldsymbol{C}}_b^n=\boldsymbol{C}_b^n(\boldsymbol{\omega}_{nb}^b\times)=\boldsymbol{C}_b^n\left[(\boldsymbol{\omega}_{ib}^b-\boldsymbol{\omega}_{ie}^b-\boldsymbol{\omega}_{en}^b)\times\right]=\boldsymbol{C}_b^n\left[(\boldsymbol{\omega}_{ib}^b-\boldsymbol{\omega}_{ie}^b)\times\right]\\ \dot{\boldsymbol{v}}^n=\boldsymbol{C}_b^n\boldsymbol{f}_{sf}^b+\boldsymbol{g}^n\end{cases}$$

与其对应的简化误差传播方程 $\begin{cases}\dot{\boldsymbol{\phi}}=\boldsymbol{\phi}\times\boldsymbol{\omega}_{ie}^n-\boldsymbol{\varepsilon}^n\\ \delta\dot{\boldsymbol{v}}^n=\boldsymbol{f}_{sf}^n\times\boldsymbol{\phi}+\boldsymbol{\nabla}^n\end{cases}$

建立状态空间模型 $\begin{cases}\dot{\boldsymbol{X}}=\boldsymbol{F}\boldsymbol{X}+\boldsymbol{W}\\ \boldsymbol{Z}=\delta\boldsymbol{v}^n=\boldsymbol{H}\boldsymbol{X}+\boldsymbol{V}\end{cases}$

$$\boldsymbol{X}=\begin{bmatrix}\phi_E & \phi_N & \phi_U & \delta v_E & \delta v_N & \varepsilon_E & \varepsilon_N & \varepsilon_U & \nabla_E & \nabla_N\end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{F}=\begin{bmatrix}0 & \omega_U & -\omega_N & 0 & 0 & -1 & 0 & 0 & 0 & 0\\ -\omega_U & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0\\ \omega_N & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0\\ 0 & -g & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ \multicolumn{10}{c}{\mathbf{0}_{5\times10}}\end{bmatrix} \qquad \boldsymbol{H}=\begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

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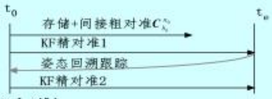


## 5 捷联惯导初始对准

5.5 不同基座环境下初始对准方法的选择和讨论

（1）纯**静基座**下，解析对准可直接当作精对准用，其结果是最优的(没有之一)，因其几乎不受加速度偏值斜坡的影响； → 理论值上限

（2）**晃动基座**下，“间接粗对准+Kalman滤波精对准+数据存储+回溯”是地球人都知道的本世纪最好的初始对准方法(时间短、精度高)，实际应用中大失准角对准问题完全没必要；

（3）**多位置**对准方法，可有效辨识出陀螺常值零偏和加速度计零偏，提高初始对准精度；

（4）连续**旋转**调制对准方法，可提高初始对准精度，其只是减弱了陀螺零偏的影响，但不能将其辨识出来；

（5）在(1)和(2)中，等效东向陀螺**常值零偏**是影响方位对准的主要因素；在(3)和(4)中，陀螺角度**随机游走**是主要因素，但由于转动动态，可能会诱发一些其它误差源，需要细心建模补偿，否则对准结果不一定理想。

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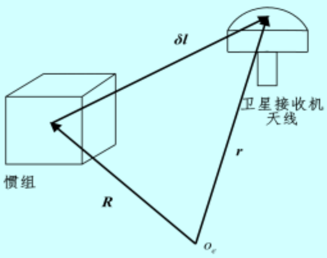
## 6 SINS/GPS组合导航




（1）空间不同步问题（杆臂误差）

卫导天线地心矢量与惯导地心矢量关系

$$\boldsymbol{r} = \boldsymbol{R} + \delta\boldsymbol{l}$$

两边相对e系求导，得

$$\left.\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}\right|_e = \left.\frac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}t}\right|_e + \left.\frac{\mathrm{d}(\delta\boldsymbol{l})}{\mathrm{d}t}\right|_e = \left.\frac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}t}\right|_e + \left.\frac{\mathrm{d}(\delta\boldsymbol{l})}{\mathrm{d}t}\right|_b + \boldsymbol{\omega}_{eb} \times \delta\boldsymbol{l} = \left.\frac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}t}\right|_e + \boldsymbol{\omega}_{eb} \times \delta\boldsymbol{l}$$

在n系投影，得

$$\boldsymbol{v}_{\mathrm{GNSS}}^n = \boldsymbol{v}_{\mathrm{INS}}^n + \boldsymbol{C}_b^n(\boldsymbol{\omega}_{eb}^b \times \delta\boldsymbol{l}^b)$$

由此可得速度误差

$$\delta\boldsymbol{v}_{\mathrm{L}}^n = \boldsymbol{v}_{\mathrm{INS}}^n - \boldsymbol{v}_{\mathrm{GNSS}}^n = -\boldsymbol{C}_b^n(\boldsymbol{\omega}_{eb}^b \times \delta\boldsymbol{l}^b) = -\boldsymbol{C}_b^n(\boldsymbol{\omega}_{eb}^b \times)\delta\boldsymbol{l}^b$$

易得位置误差 $\delta\boldsymbol{p}_{\mathrm{GL}} = \boldsymbol{p}_{\mathrm{INS}} - \boldsymbol{p}_{\mathrm{GNSS}} = -\boldsymbol{M}_{pv}\boldsymbol{C}_b^n\delta\boldsymbol{l}^b$

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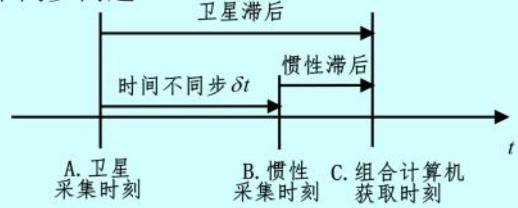
# 6 SINS/GPS组合导航

（2）时间不同步问题




卫导速度与惯导速度之间关系

$$\boldsymbol{v}_{\mathrm{GNSS}}^{n} + \boldsymbol{a}^{n}\delta t = \boldsymbol{v}_{\mathrm{INS}}^{n} \qquad \boldsymbol{a}^{n} \approx \frac{\boldsymbol{v}_{\mathrm{INS}(m)}^{n} - \boldsymbol{v}_{\mathrm{INS}(m-1)}^{n}}{T}$$

易得速度误差和位置误差

$$\delta\boldsymbol{v}_{\delta t}^{n} = \boldsymbol{v}_{\mathrm{INS}}^{n} - \boldsymbol{v}_{\mathrm{GNSS}}^{n} = \boldsymbol{a}^{n}\delta t$$

$$\delta\boldsymbol{p}_{\delta t} = \boldsymbol{p}_{\mathrm{INS}} - \boldsymbol{p}_{\mathrm{GNSS}} = \boldsymbol{M}_{pv}\boldsymbol{v}_{\mathrm{INS}}^{n}\delta t$$

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# 6 SINS/GPS组合导航

（3）组合导航状态空间模型

状态空间模型
$$\begin{cases} \dot{\boldsymbol{X}} = \boldsymbol{F}\boldsymbol{X} + \boldsymbol{G}\boldsymbol{W}^{b} \\ \boldsymbol{Z} = \begin{bmatrix} \boldsymbol{v}_{\mathrm{INS}}^{n} - \boldsymbol{v}_{\mathrm{GNSS}}^{n} \\ \boldsymbol{p}_{\mathrm{INS}} - \boldsymbol{p}_{\mathrm{GNSS}} \end{bmatrix} = \boldsymbol{H}\boldsymbol{X} + \boldsymbol{V} \end{cases}$$

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{\phi}^{\mathrm{T}} & (\delta\boldsymbol{v}^{n})^{\mathrm{T}} & (\delta\boldsymbol{p})^{\mathrm{T}} & (\boldsymbol{\varepsilon}^{b})^{\mathrm{T}} & (\nabla^{b})^{\mathrm{T}} & \underline{(\delta\boldsymbol{l}^{b})^{\mathrm{T}} \quad \delta t} \end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{M}_{aa} & \boldsymbol{M}_{av} & \boldsymbol{M}_{ap} & -\boldsymbol{C}_{b}^{n} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times4} \\ \boldsymbol{M}_{va} & \boldsymbol{M}_{vv} & \boldsymbol{M}_{vp} & \boldsymbol{0}_{3\times3} & \boldsymbol{C}_{b}^{n} & \boldsymbol{0}_{3\times4} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{M}_{pv} & \boldsymbol{M}_{pp} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times4} \\ & & & \boldsymbol{0}_{10\times19} & & \end{bmatrix}$$

尽量已知补偿
不列入模型

量测中得出

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{0}_{3\times3} & \boldsymbol{I}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times6} & -\boldsymbol{C}_{b}^{n}(\boldsymbol{\omega}_{eb}^{b}\times) & \boldsymbol{a}^{n} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{I}_{3\times3} & \boldsymbol{0}_{3\times6} & -\boldsymbol{M}_{pv}\boldsymbol{C}_{b}^{n} & \boldsymbol{M}_{pv}\boldsymbol{v}^{n} \end{bmatrix}$$

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## 7 捷联惯导算法调试要点讨论

（1）原始数据采集、传输和存储不得有误码，采用累积增量数据格式可以有效避免偶然的传输误码；
（2）6只惯性传感器数据采集须考虑时间同步性问题（Time-asynchrony identification between inertial sensors in SIMU,2015）；此外，还须考虑加速度计内杆臂实验标定或直接测量补偿（捷联惯导系统内杆臂补偿方法及试验验证,2012）；
（3）除非像激光陀螺的抖动解调须FIR滤波，其它采用比如IIR、KF、小波、神经网络之类的数据降噪预处理方式几乎不会带来任何好处，万一需要，简单采用高频采样再滑动平均输出就是最好的方式；
（4）Allan方差是分析陀螺仪主要性能——稳定性的最简单最实用方法（http://blog.sina.com.cn/s/blog_40edfdc90102y1ar.html），加速度计可直接采用“t-1s平均”时域图直观分析；
（5）导航性能大部分归功于好的SIMU标定（惯性仪器测试与数据分析第9章,2012），反之标定不好会引起后续动态环境下导航的许多问题；

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## 7 捷联惯导算法调试要点讨论

（6）实验室静基座下进行解析对准，进入纯惯性导航后再绕方位转180°，导航1h的定位精度实验，基本可以反映系统的定位精度和标定性能；
（7）长航时、大跨域、高精度惯导系统应当重视实际重力倾斜的影响；
（8）SINS与GNSS之间最好通过硬件PPS同步，否则采用NMEA0183协议之类串口传输存在不固定延时，在高速高动态下容易引起额外误差；在低精度SINS/GNSS中杆臂误差很难估计准确；
（9）组合导航采用最简单的方法就是最好的方法(实用、可靠性高、精度也不差)，线性模型+Kalman滤波+细节处理足矣（传统组合导航中的实用Kalman滤波技术评述,2020）；
（10）实践(实验)是检验真理(算法)的唯一标准！

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